# A SAT Solver's Opinion on the Erdős-Faber-Lovász Conjecture

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SAT 2023
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# Outline

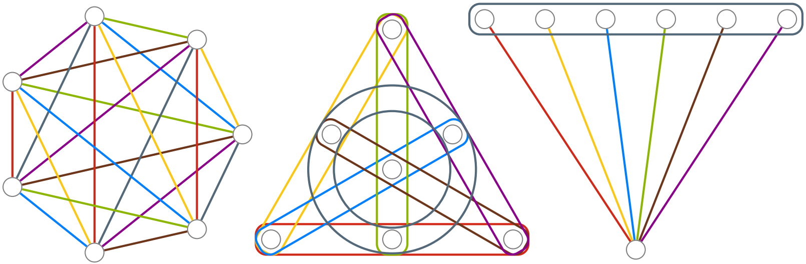
Mathematician → Conjecture → Mathematician → Time-out → SAT solver

**In this talk**

What conjecture

How we tackle it

# Erdős, Faber, Lovász



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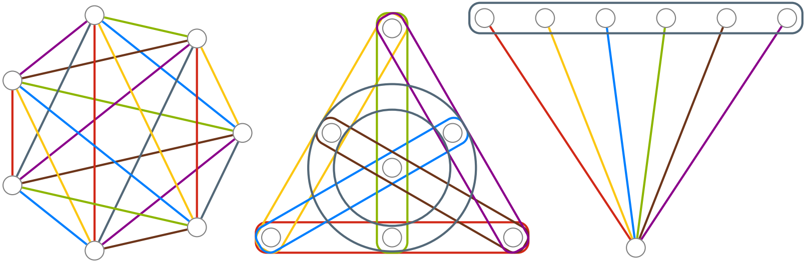

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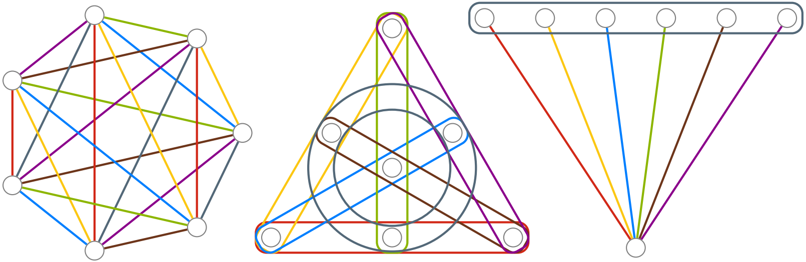

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## Theorem ([KKK+21])

*There is $n_0$ s.t. when $n > n_0$, every linear hypergraph with $n$ vertices is $n$-edge-colorable.*

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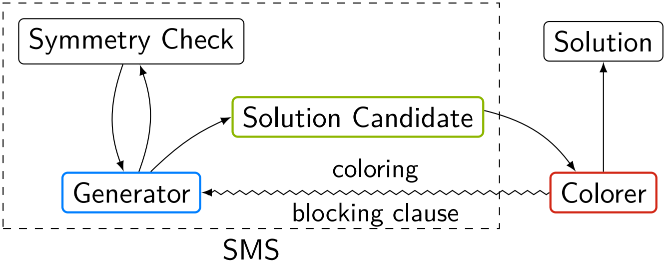
- we also verify a generalized conjecture and generate extremal hypergraphs (which need $n$ colors);
- inspired by the generated examples, we prove some general theorems about (non-)extremality;

# Overview of “∼SAT solver”

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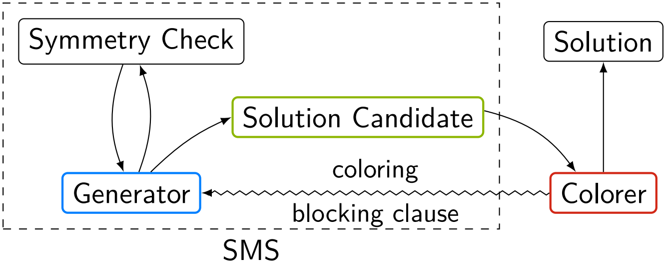


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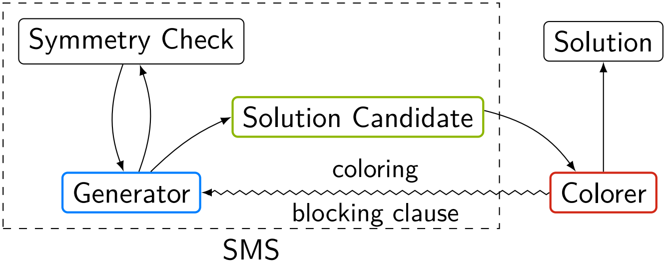


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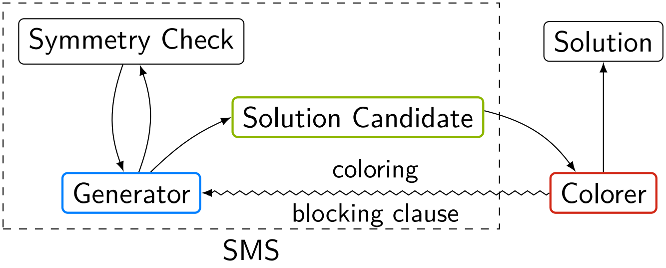


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- SMS relies on a custom propagator (in Cadical), the coloring is "incremental".

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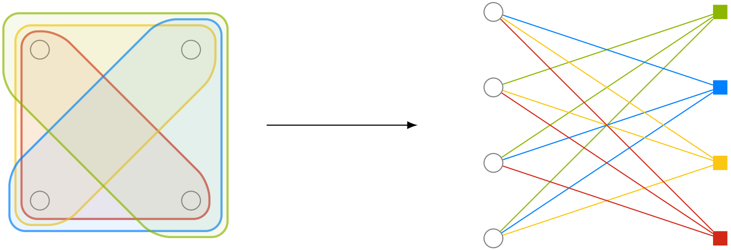

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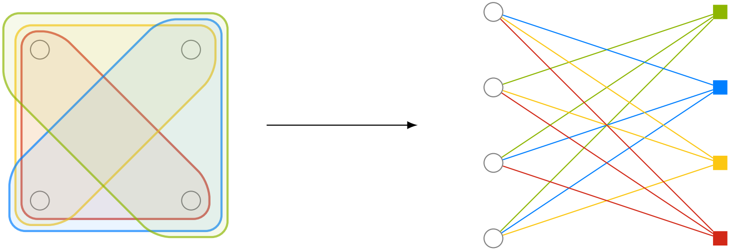

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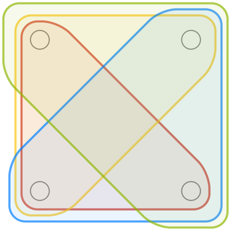

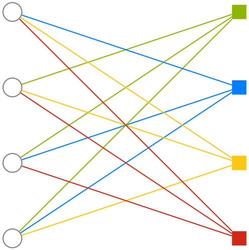

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- linearity $=$ no 4-cycle in the incidence graph.

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- follows the co-certificate learning method [KPS23];
- can also generate non-$(n-1)$-colorable (extremal) hypergraphs in the same manner;

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**Conjecture (Füredi [Für86], Berge [Ber89])**

Let $H = (V, E)$ be a linear hypergraph, and let $\zeta = \max_{v \in V} \left|\bigcup_{v \in e} e\right|$. Then $H$ is $\zeta$-edge-colorable. (For graphs $\zeta = \Delta + 1$.)

# Results

# EFL verification

**Theorem**

*The EFL' Conjecture holds for $n, m$ if $n \leq 12$; or*

- $n = 13$ *and* $m \in [13, 32] \cup [55, 78]$*; or*
- $n = 14$ *and* $m \in [14, 28] \cup [70, 91]$*; or*
- $n = 15$ *and* $m \in [15, 29] \cup [84, 105]$*; or*
- $n = 16$ *and* $m \in [16, 30] \cup \{99\} \cup [101, 120]$*; or*
- $n = 17$ *and* $m \in [17, 30] \cup [117, 136]$*; or*
- $n = 18$ *and* $m \in [18, 31] \cup [134, 153]$.

# Time

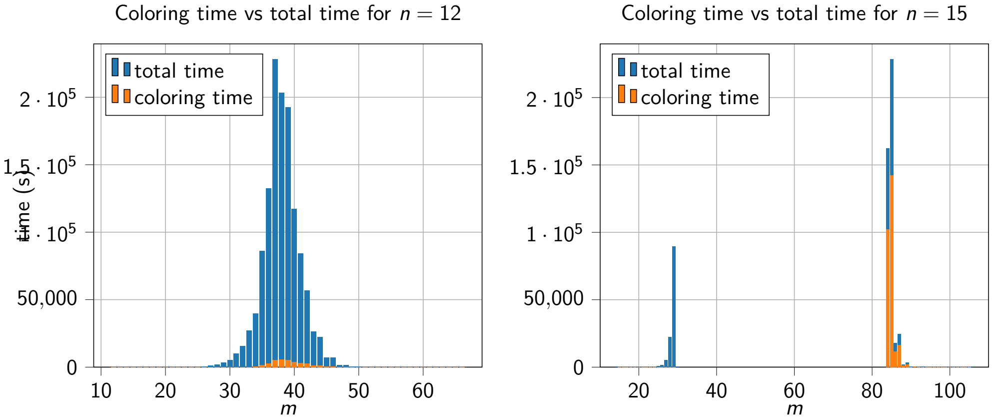



Figure: The distribution of the running times for $n = 12$ and $n = 15$.

# FB verification

**Theorem**

*The FB' Conjecture holds for $n$ and $m$ if $n \leq 10$; or*

- $n = 11$ *and* $m \in [11, 20] \cup [37, 55]$*; or*
- $n = 12$ *and* $m \in [12, 17] \cup [49, 66]$.

# Time

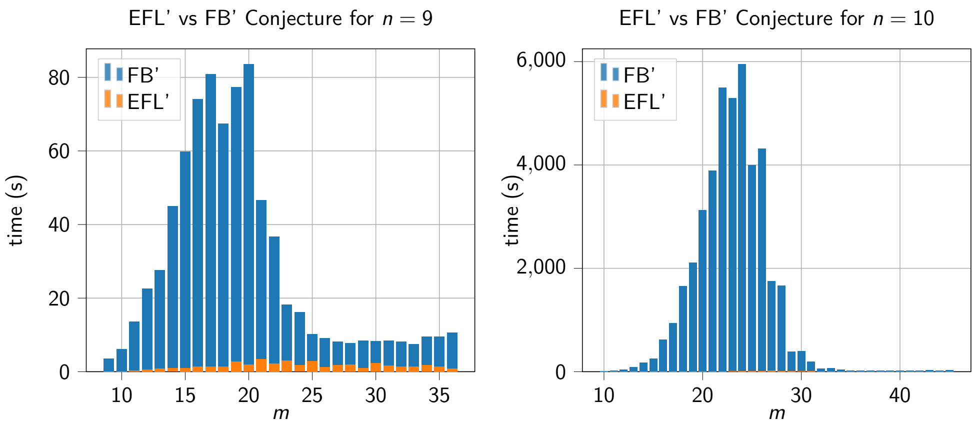



Figure: The distribution of the running times for the EFL' and FB' conjectures for $n \in [9, 10]$.

# Extremal hypergraphs

**Theorem**

*The linear spaces with $n \leq 12$ vertices and chromatic index $n$ are precisely $\mathcal{H}_{n,k}$ for all $k \not\equiv n$ mod 2, and additionally $\mathcal{H}_{7,3}, \mathcal{H}_{9,3}, \mathcal{H}_{11,3}$, $\mathcal{H}_{7,3\not\cap 3}$, $\mathcal{H}_{11,3\cap 3}$, $\mathcal{H}_{11,3\not\cap 3}$, and the Fano plane.*

# Chain

$K_n$ is extremal precisely when $n$ is odd

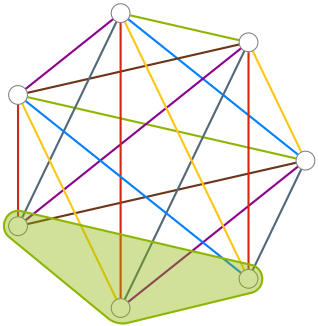
| $n$ | 5 | 6 | 7 | 8 | 9 | 10 | 11 |
|---|---|---|---|---|---|---|---|
| | ✓ | ✗ | ✓ | ✗ | ✓ | ✗ | ✓ |

# Chain

$\mathcal{H}_{n,2}$ is extremal precisely when $n$ is odd

| $n$ | 5 | 6 | 7 | 8 | 9 | 10 | 11 |
|---|---|---|---|---|---|---|---|
| $k$ | | | | | | | |
| 2 | ✓ | ✗ | ✓ | ✗ | ✓ | ✗ | ✓ |

$\mathcal{H}_{n,k}$ is the linear space with one $k$-edge and all the rest 2-edges.

$\mathcal{H}_{7,3} =$ 

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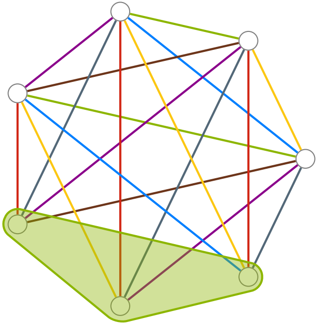
$\mathcal{H}_{7,3} =$ 

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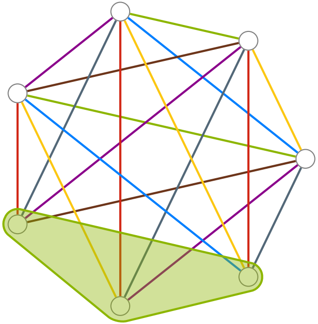
$\mathcal{H}_{7,3} =$ 

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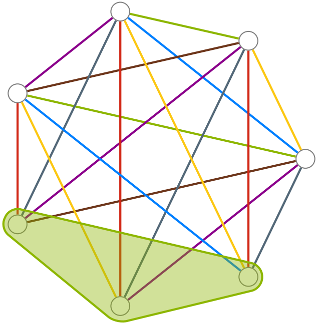

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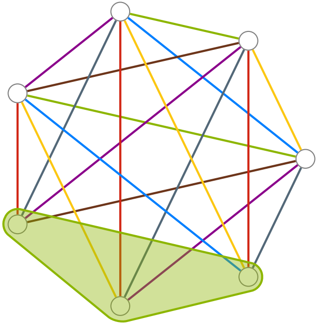
$\mathcal{H}_{7,3} =$ 

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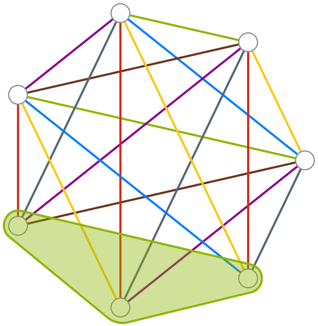

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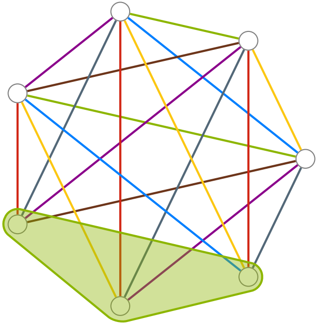
$\mathcal{H}_{7,3} =$ 

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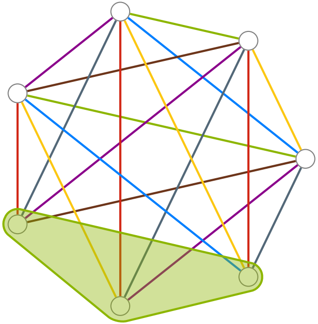
$\mathcal{H}_{7,3} =$ 

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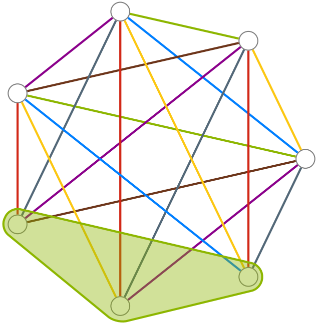

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$\mathcal{H}_{n,k}$ is extremal when $n \not\equiv k \mod 2$, and ...

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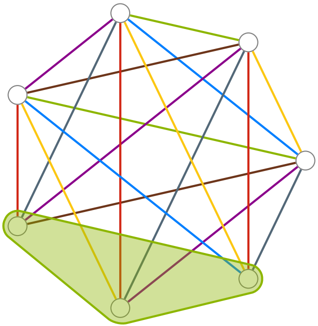
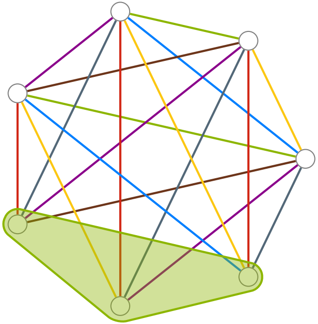


# Chain

$\mathcal{H}_{n,k}$ is extremal when $n \not\equiv k \mod 2$, and sometimes when $n \equiv k \equiv 1 \mod 2$

| $k$ \ $n$ | 5 | 6 | 7 | 8 | 9 | 10 | 11 |
|---|---|---|---|---|---|---|---|
| 2 | ✓ | ✗ | ✓ | ✗ | ✓ | ✗ | ✓ |
| 3 | ✗ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ |
| 4 | ✓ | ✗ | ✓ | ✗ | ✓ | ✗ | ✓ |
| 5 | | ✓ | ✗ | ✓ | ✗ | ✓ | ✗ |
| 6 | | | ✓ | ✗ | ✓ | ✗ | ✓ |
| 7 | | | | ✓ | ✗ | ✓ | ✗ |
| 8 | | | | | ✓ | ✗ | ✓ |
| 9 | | | | | | ✓ | ✗ |
| 10 | | | | | | | ✓ |

$\mathcal{H}_{n,k}$ is the linear space with one $k$-edge and all the rest 2-edges.

$\mathcal{H}_{7,3} =$ 

$\mathcal{H}_{n,k}$ is extremal when $n \not\equiv k \mod 2$, and sometimes when $n \equiv k \equiv 1 \mod 2$, but never when $n \equiv k \equiv 0 \mod 2$.

| $n$ | 5 | 6 | 7 | 8 | 9 | 10 | 11 |
|---|---|---|---|---|---|---|---|
| $k$ | | | | | | | |
| 2 | ✓ | ✗ | ✓ | ✗ | ✓ | ✗ | ✓ |
| 3 | ✗ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ |
| 4 | ✓ | ✗ | ✓ | ✗ | ✓ | ✗ | ✓ |
| 5 | | ✓ | ✗ | ✓ | ✗ | ✓ | ✗ |
| 6 | | | ✓ | ✗ | ✓ | ✗ | ✓ |
| 7 | | | | ✓ | ✗ | ✓ | ✗ |
| 8 | | | | | ✓ | ✗ | ✓ |
| 9 | | | | | | ✓ | ✗ |
| 10 | | | | | | | ✓ |

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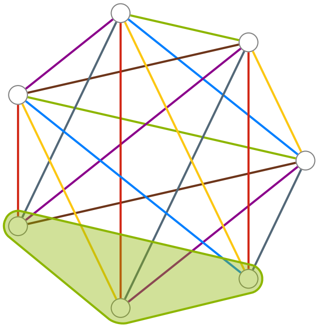
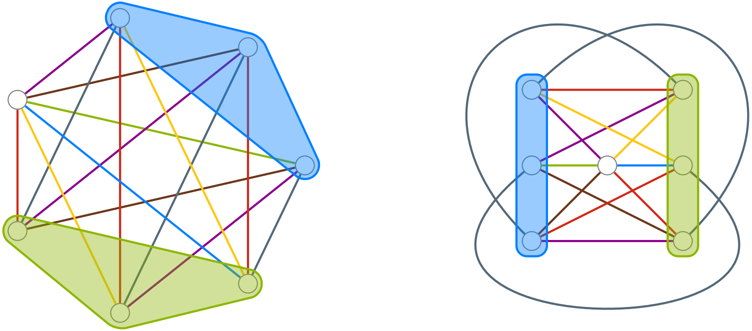
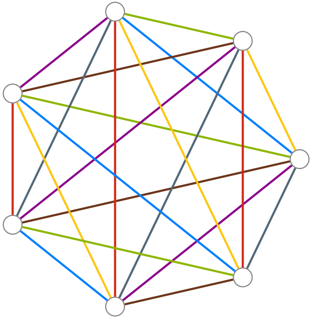
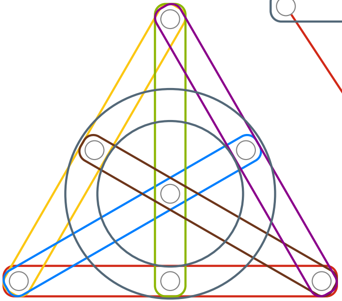
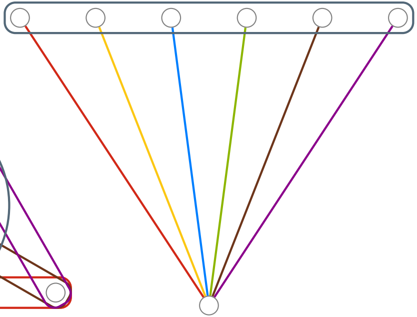
$\mathcal{H}_{7,3} =$ 

# More nice pictures

$\mathcal{H}_{7,3\not\cap 3}$



# More nice pictures

# Conclusion

- SMS is great for (hyper)graph search with constraints, even for problems with a coNP-complete constraint; try it out:
  - `https://sat-modulo-symmetries.readthedocs.io`
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- a SAT-based exploration of small cases can inspire general results;
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- also hard for coloring, in particular when not colorable (extremal);

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