CADIBACK

Extracting Backbones with CaDiCaL

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Backbone

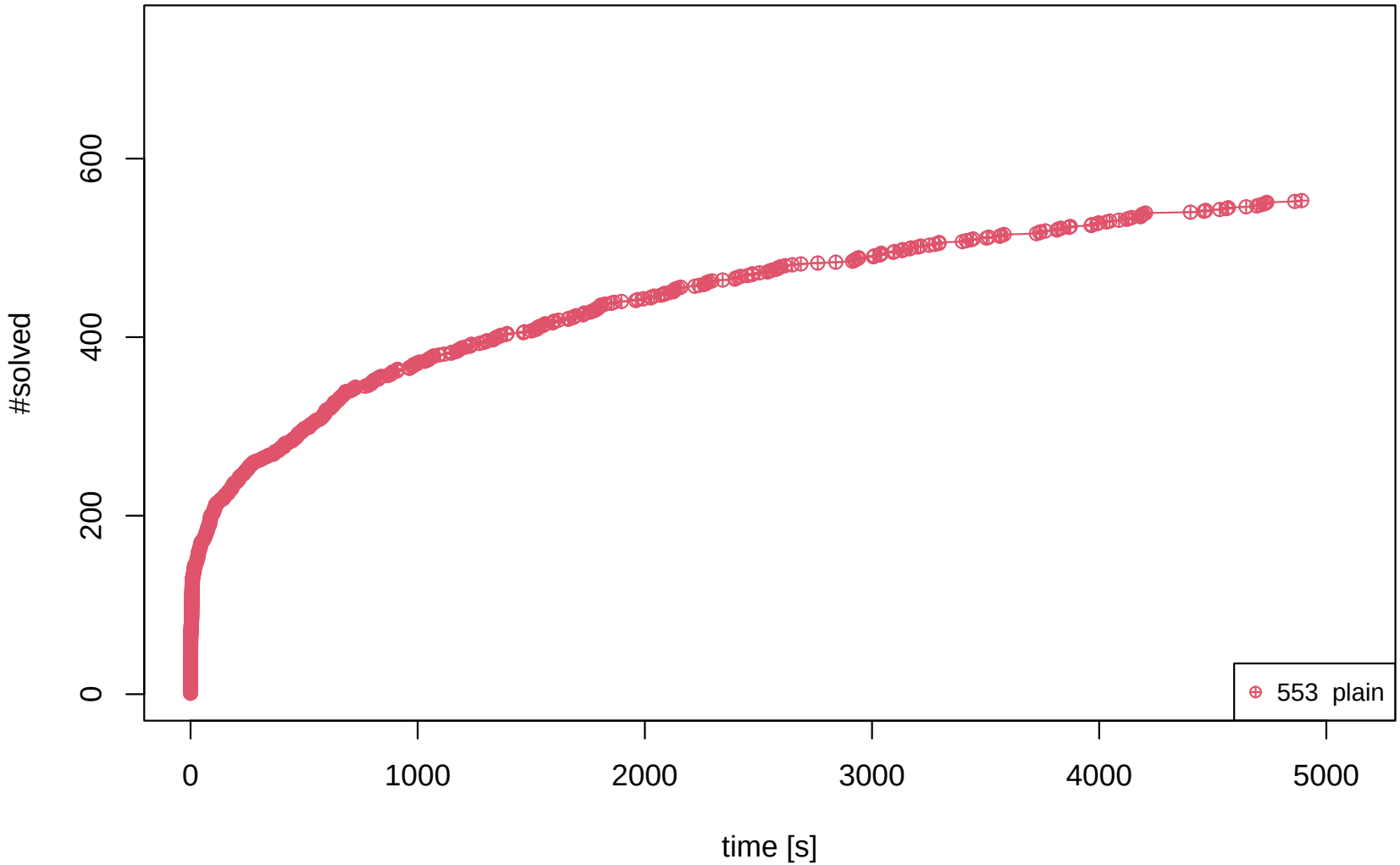
The backbone of a satisfiable formula is the set of literals that are true in all satisfying assignments.

$$a \quad \wedge \quad \overline{a} \vee b \quad \wedge \quad \overline{a} \vee \overline{b} \vee c \quad \wedge$$
$$d \vee e \vee f \quad \wedge \quad \overline{d} \vee e \vee f \quad \wedge \quad d \vee \overline{e} \vee f \quad \wedge \quad \overline{d} \vee \overline{e} \vee f \quad \wedge$$
$$\mathrm{PHP} \vee g$$

Backbone Extraction

backbone (CNF φ)

1 $(res, \sigma) \leftarrow \mathsf{SAT}(\varphi)$
2 $\mathcal{B} \leftarrow \emptyset, \quad \Lambda \leftarrow \sigma$
3 **while** $\Lambda \neq \emptyset$ **do**
4 $\quad \ell \leftarrow$ pick literal from Λ
5 $\quad (res, \sigma) \leftarrow \mathsf{SAT}(\varphi \mid \neg\ell)$
6 $\quad$ **if** res **then** // satisfiable
7 $\quad\quad \Lambda \leftarrow \Lambda \setminus \{\ell\}$
8 $\quad$ **else** // unsatisfiable
9 $\quad\quad \mathcal{B} \leftarrow \mathcal{B} \cup \{\ell\}$
10 $\quad\quad \Lambda \leftarrow \Lambda \setminus \{\ell\}$
11 **return** $\mathcal{B}$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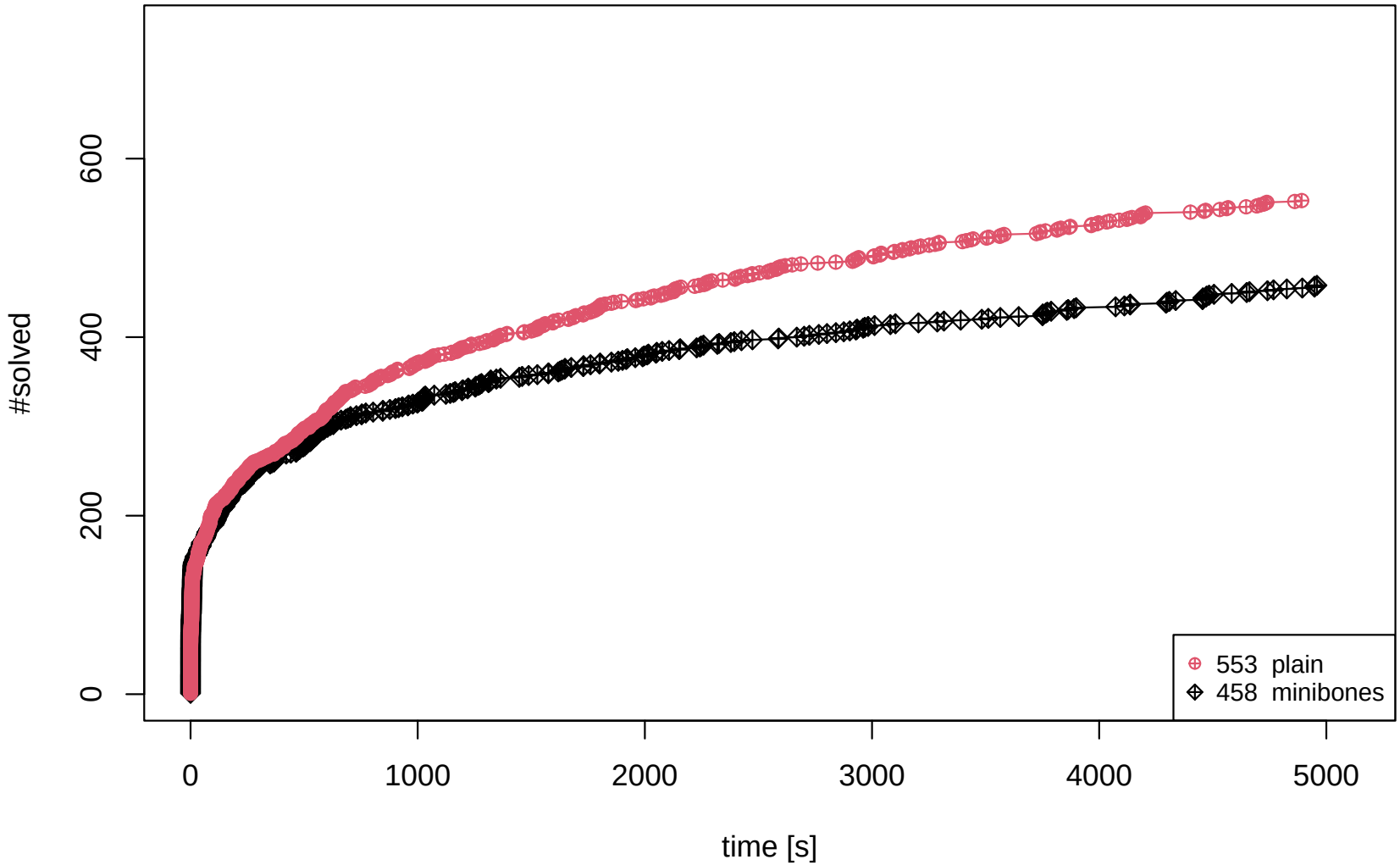
Evaluation

- Benchmarks
  - SAT competition 2004 – 2022
  - remove comments, pretty print, compute hash, remove duplicates
  - 5,000 second of Kissat 3.0.0 [Biere and Fleury, 2022]
  - total 1798 benchmarks (6 GB)
  - `https://cca.informatik.uni-freiburg.de/sc04to22sat.zip`
- Baseline
  - MiniBones: Mikolás Janota, Inês Lynce, João Marques-Silva [Janota et al., 2015]
  - `-u -c 100 -i` slightly outperformed default configuration
  - ported to newer C++ compiler
  - `https://github.com/arminbiere/minibones`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Antithesis 1: Incremental SAT solving did not improve

"Our results, obtained on a number of well-known practically significant applications, suggest that most improvements made to SAT solvers in recent years have no positive impact on the overall performance when solvers are used incrementally."

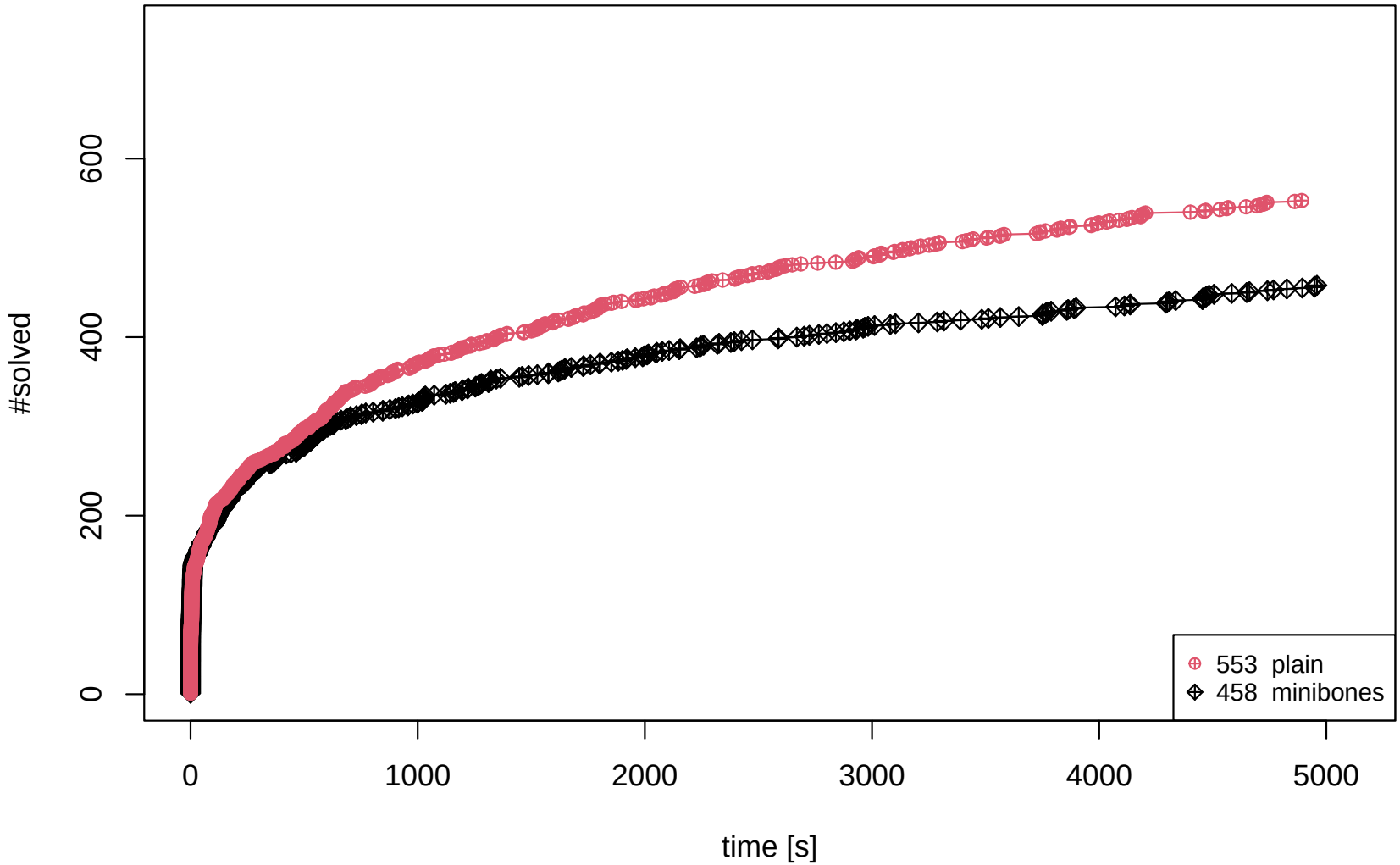
Assessing Progress in SAT Solvers Through the Lens of Incremental SAT
– Stepan Kochemazov, Alexey Ignatiev, João Marques-Silva
[Kochemazov et al., 2021]

"My experience at Intel: no progress on industrial optimization problems"

Introducing Intel® SAT solver Talk at SAT'22
– Alexander Nadel
[Nadel, 2022]

"In my world there has been no progress since MiniSAT 2.2"

Discussion of the SAT Museum PoS'23
– Oliver Kullmann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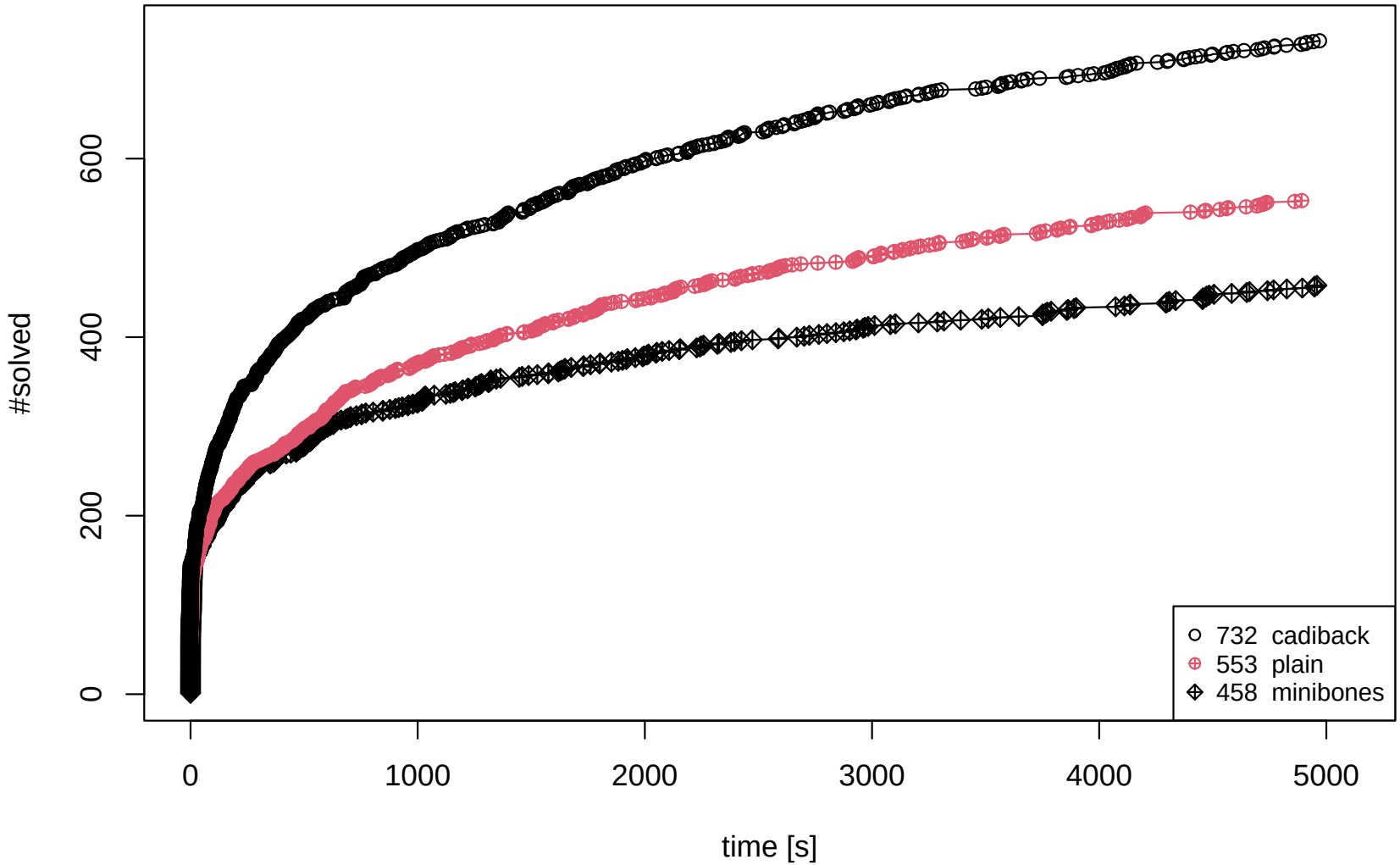
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458 minibones

Backbone Extraction

backbone (CNF φ)

1 $(res, \sigma) \leftarrow \mathsf{SAT}(\varphi)$

2 $\mathcal{B} \leftarrow \emptyset, \quad \Lambda \leftarrow \sigma$

3 **while** $\Lambda \neq \emptyset$ **do**

4 $\quad \ell \leftarrow$ pick literal from Λ

5 $\quad (res, \sigma) \leftarrow \mathsf{SAT}(\varphi \mid \neg\ell)$

6 $\quad$ **if** res **then** // satisfiable

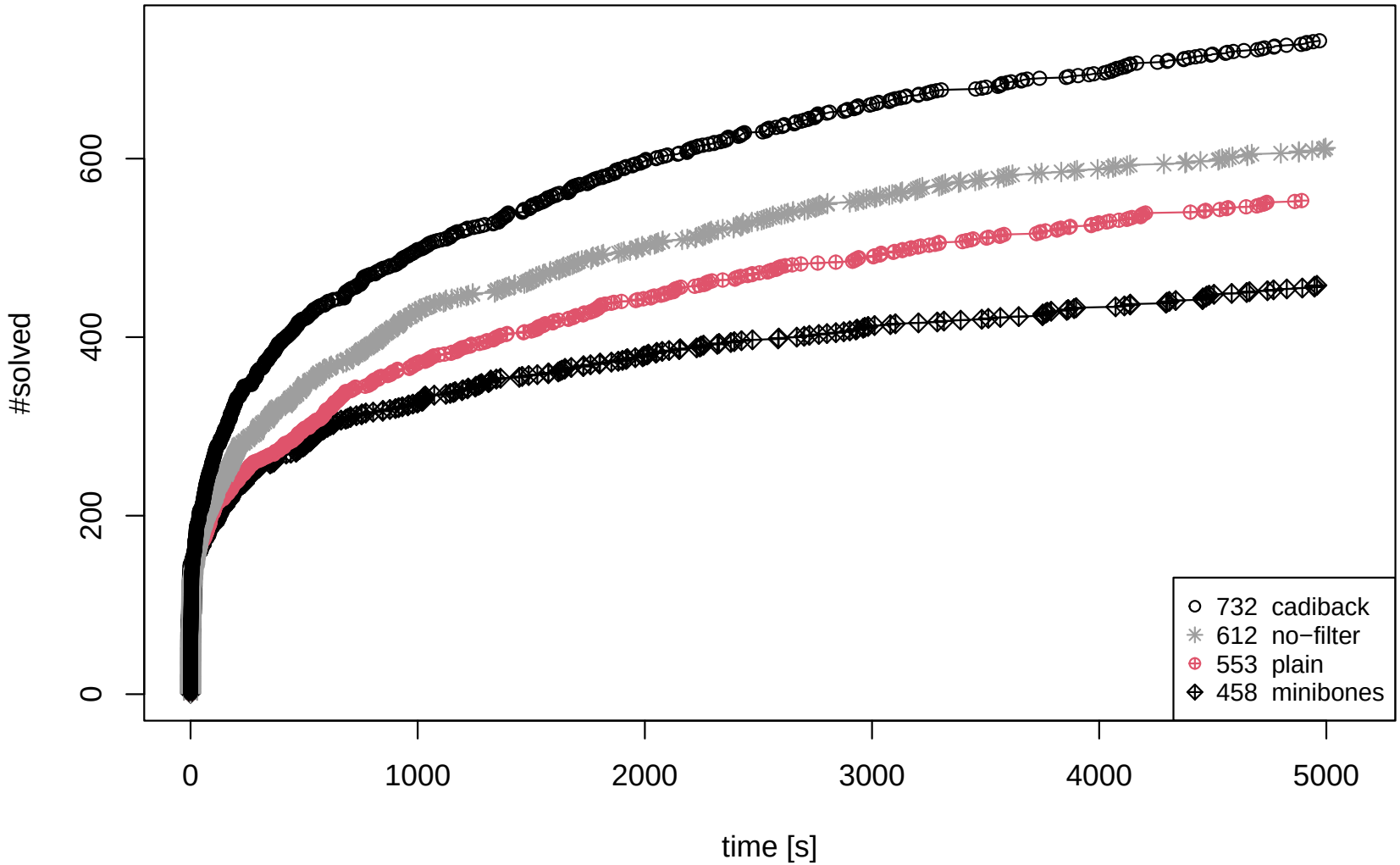
7 $\quad\quad \Lambda \leftarrow \Lambda \setminus \{\ell\}$ $\Lambda \cap \sigma$

8 $\quad$ **else** // unsatisfiable

9 $\quad\quad \mathcal{B} \leftarrow \mathcal{B} \cup \{\ell\}$

10 $\quad\quad \Lambda \leftarrow \Lambda \setminus \{\ell\}$

11 **return** $\mathcal{B}$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Backbone Extraction

backbone (CNF φ)

1 $(res, \sigma) \leftarrow \mathsf{SAT}(\varphi)$

2 $\mathcal{B} \leftarrow \emptyset, \quad \Lambda \leftarrow \sigma, \quad k \leftarrow 1$

3 **while** $\Lambda \neq \emptyset$ **do**

4 $\quad \Gamma \leftarrow$ pick k literals from Λ // chunk

5 $\quad (res, \sigma) \leftarrow \mathsf{SAT}(\varphi \mid \cancel{\neg\ell} \bigvee_{\ell \in \Gamma} \neg\ell)$

6 $\quad$ **if** res **then** // satisfiable

7 $\quad\quad \Lambda \leftarrow \Lambda \cap \sigma, \quad k \leftarrow 1$

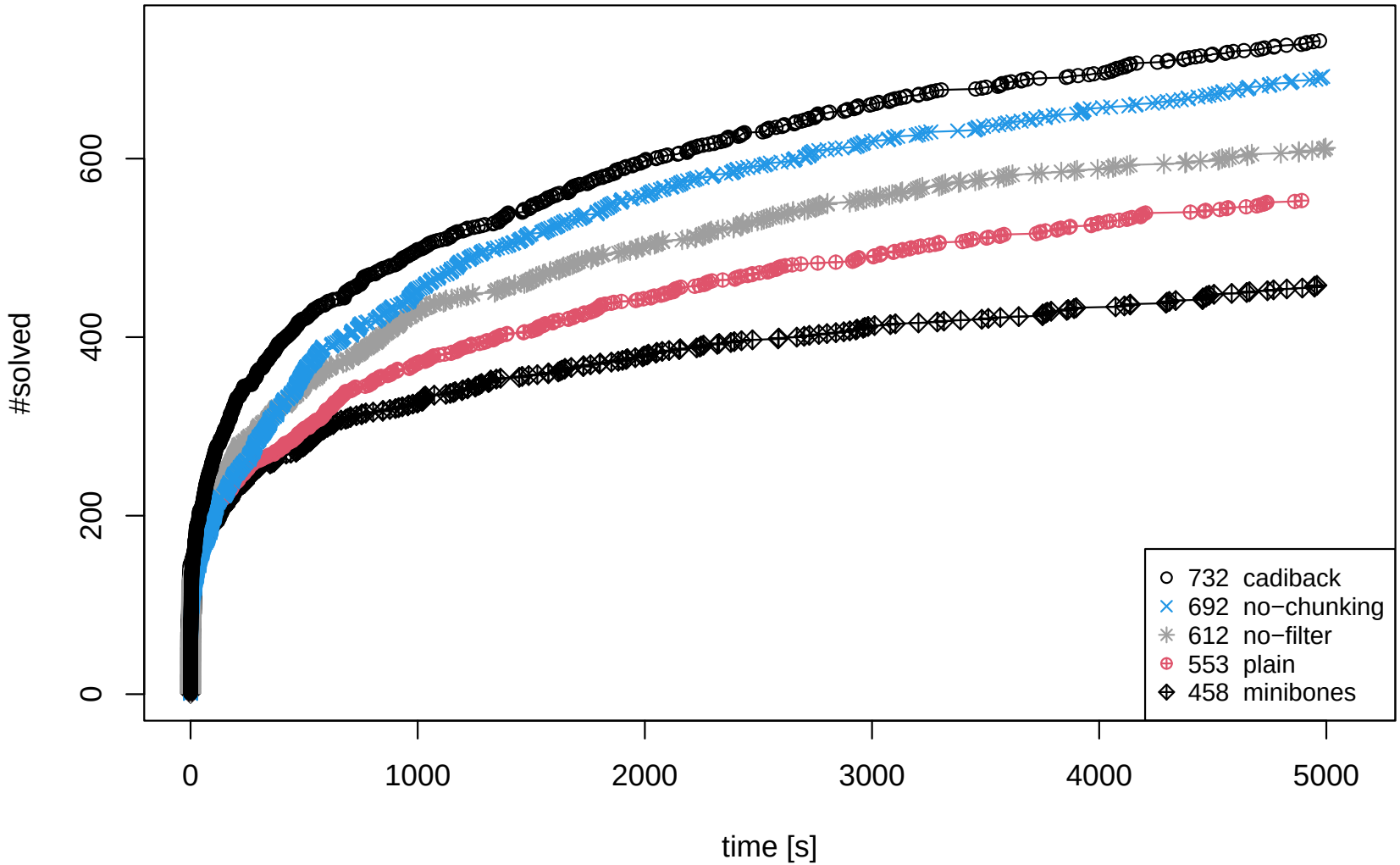
8 $\quad$ **else** // unsatisfiable

9 $\quad\quad \mathcal{B} \leftarrow \mathcal{B} \cup \cancel{\{\ell\}}\,\Gamma$

10 $\quad\quad \Lambda \leftarrow \Lambda \setminus \cancel{\{\ell\}}\,\Gamma$

11 $\quad\quad k \leftarrow |\Lambda|$

12 **return** $\mathcal{B}$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458 minibones

Constraint

$$\mathsf{SAT}(\varphi \mid \neg\ell)$$

$$\mathsf{SAT}(\varphi \mid \bigvee_{\ell\in\Gamma} \neg\ell)$$

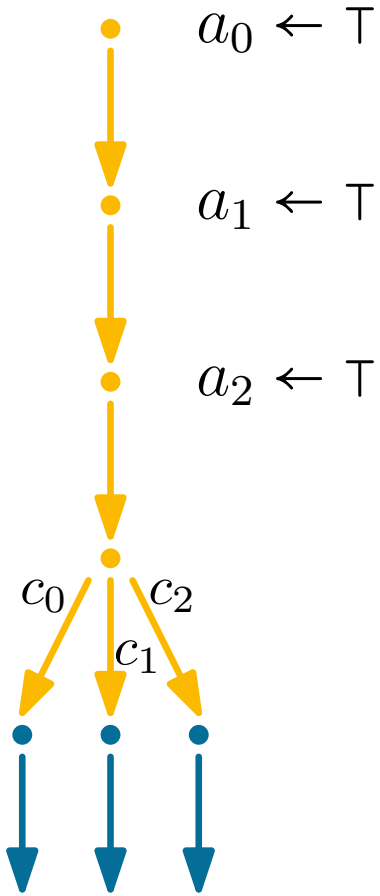
$$\underbrace{\underbrace{(\bigvee_{\ell\in\Gamma} \neg\ell \vee a)}_{\text{Clause}} \wedge \underbrace{\varphi}_{\text{Clauses}}}_{\text{Clauses}} \wedge \underbrace{\overline{a}}_{\text{Assumptions}}$$

MiniSAT [Eén and Sörensson, 2003]

$$\underbrace{(\bigvee_{\ell\in\Gamma} \neg\ell)}_{\text{Constraint}} \wedge \underbrace{\varphi}_{\text{Clauses}}$$

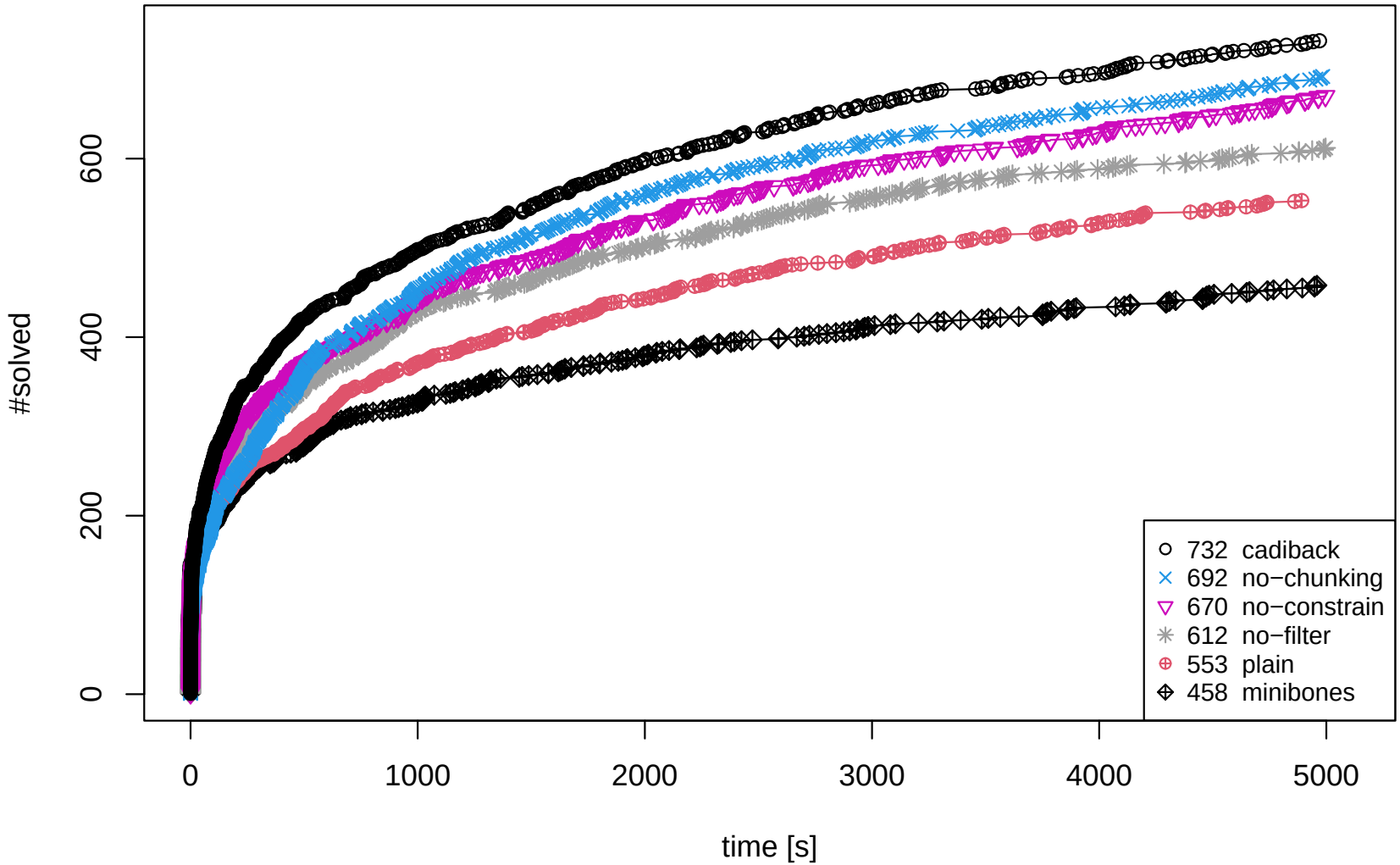
CaDiCaL [Froleyks and Biere, 2021]

Constraint



decide (constraint ρ, partial model σ)

```
1  ... // handle literal assumptions
2  if σ(ρ) = 1 then //constraint true
3      ℓ ← "first" literal in ρ with σ(ℓ)
       // speed-up future search for ℓ
4      move ℓ to the front of ρ
5  elif σ(ρ) = 0 then//constraint false
6      ... // handle conflicting constraint
7  else    // constraint undetermined
8      ℓ ← highest scored literal in σ(ρ)
9      pick ℓ as new decision and return
10 ... // fall back to default decisions
```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458 minibones

Antithesis 2: Constraints do not work

"[. . .]improvements are not very impressive"

doctored quote from a review of our rejected
SAT'21 paper on constraints
[Froleyks and Biere, 2021]

"Initially I was not sure about the argument about *clogging* with the extra clause & assumption that required constrain()"

review of this paper

Inprocessing

Incremental Inprocessing in SAT Solving [Fazekas et al., 2019]

Best student paper award SAT'19

$$\frac{\varphi\,[\rho]\,\sigma}{\varphi\,[\rho \wedge C]\,\sigma}\ \sharp$$

LEARN$^-$

$$\frac{\varphi\,[\rho \wedge C]\,\sigma}{\varphi\,[\rho]\,\sigma}$$

FORGET

$$\frac{\varphi\,[\rho \wedge C]\,\sigma}{\varphi \wedge C\,[\rho]\,\sigma}$$

STRENGTHEN

$$\frac{\varphi\,[\rho]\,\sigma}{\varphi \wedge \Delta\,[\rho]\,\sigma}\ \mathcal{I}$$

ADDCLAUSES

$$\frac{\varphi \wedge C\,[\rho]\,\sigma}{\varphi\,[\rho]\,\sigma \cdot (\omega : C)}\ \flat$$

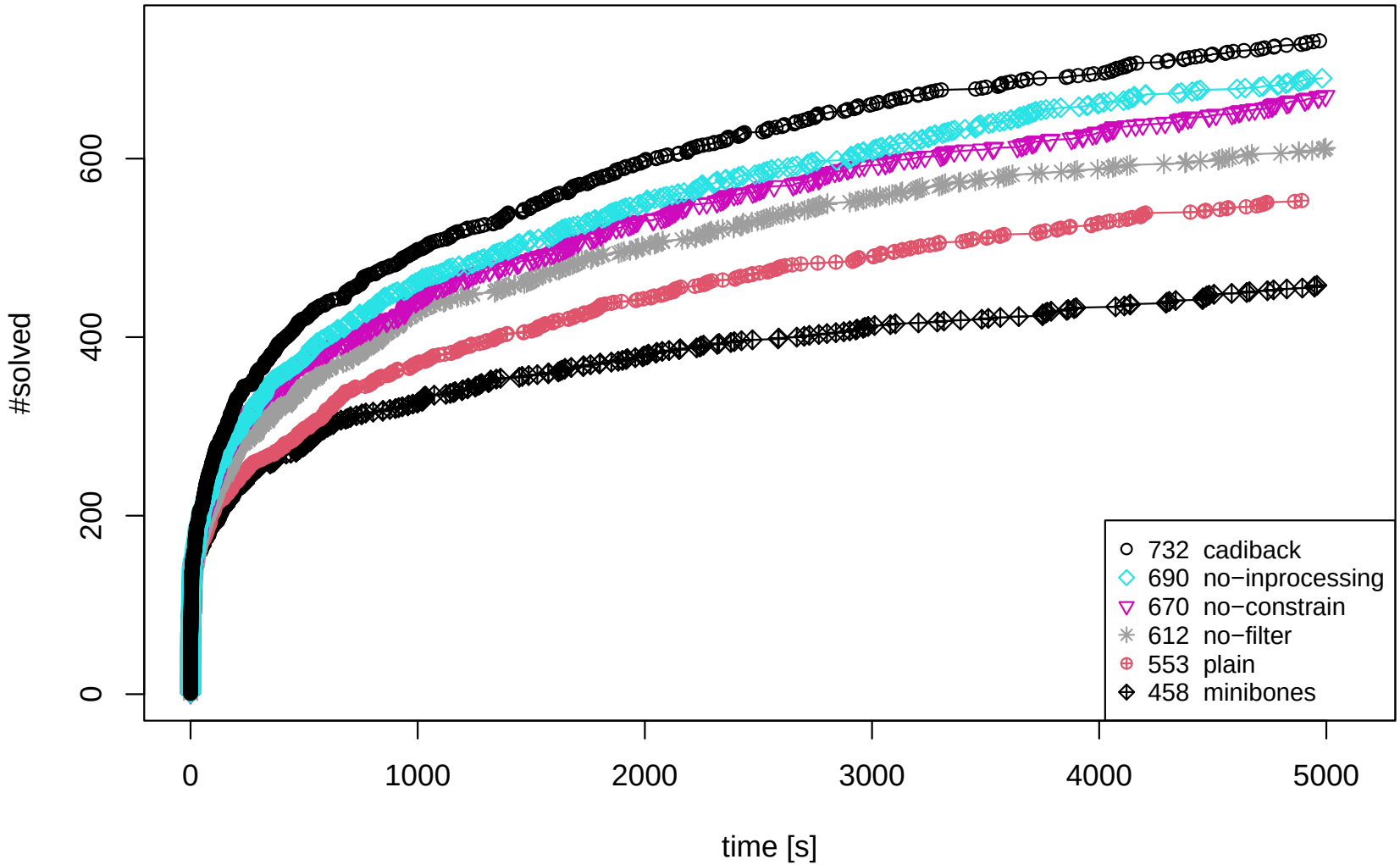
WEAKEN$^+$

$$\frac{\varphi \wedge C\,[\rho]\,\sigma}{\varphi\,[\rho]\,\sigma}\ \text{ø}$$

DROP

$$\frac{\varphi\,[\rho]\,\sigma \cdot (\omega : C) \cdot \sigma'}{\varphi \wedge C\,[\rho]\,\sigma \cdot \sigma'}\ \partial$$

RESTORE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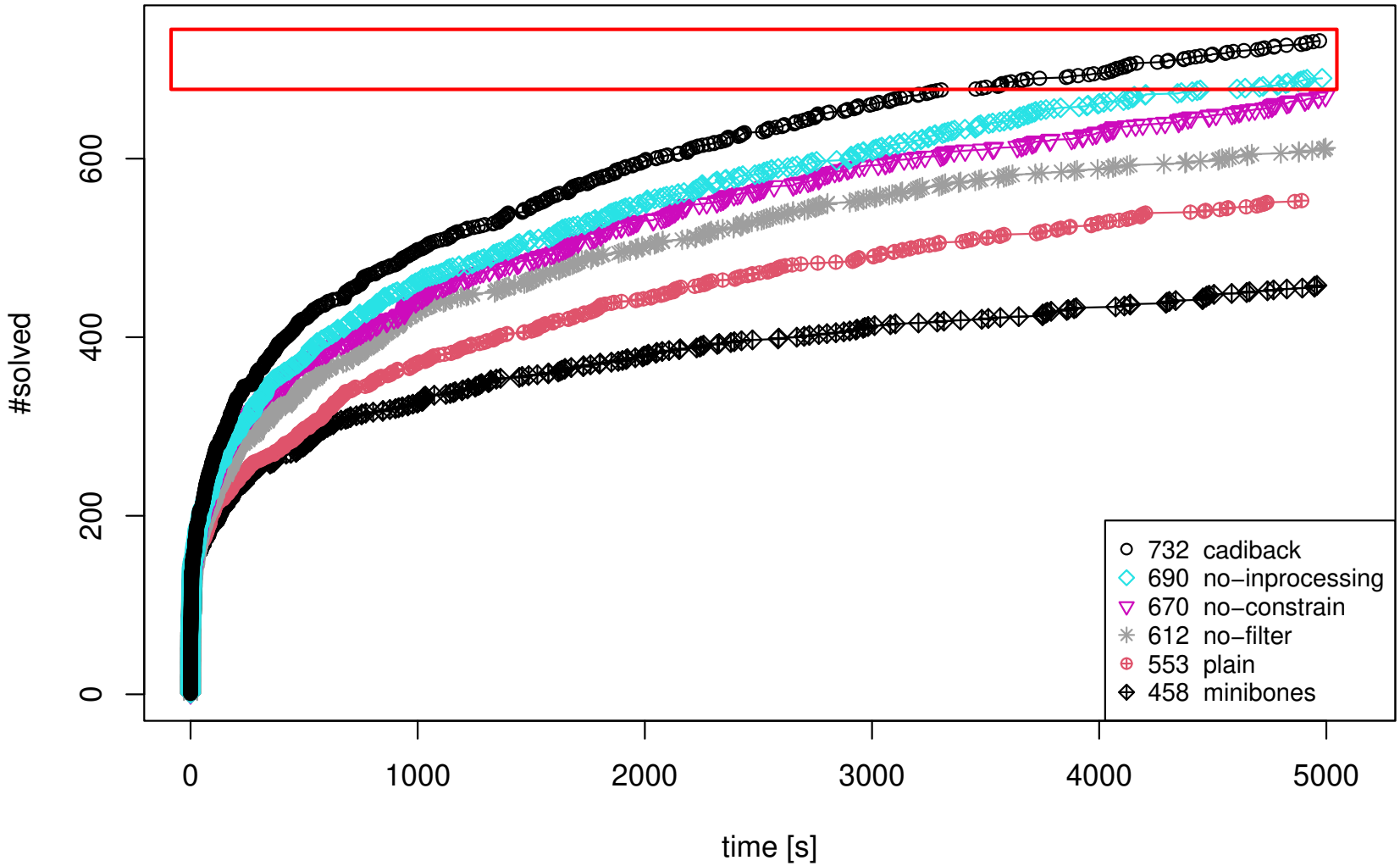
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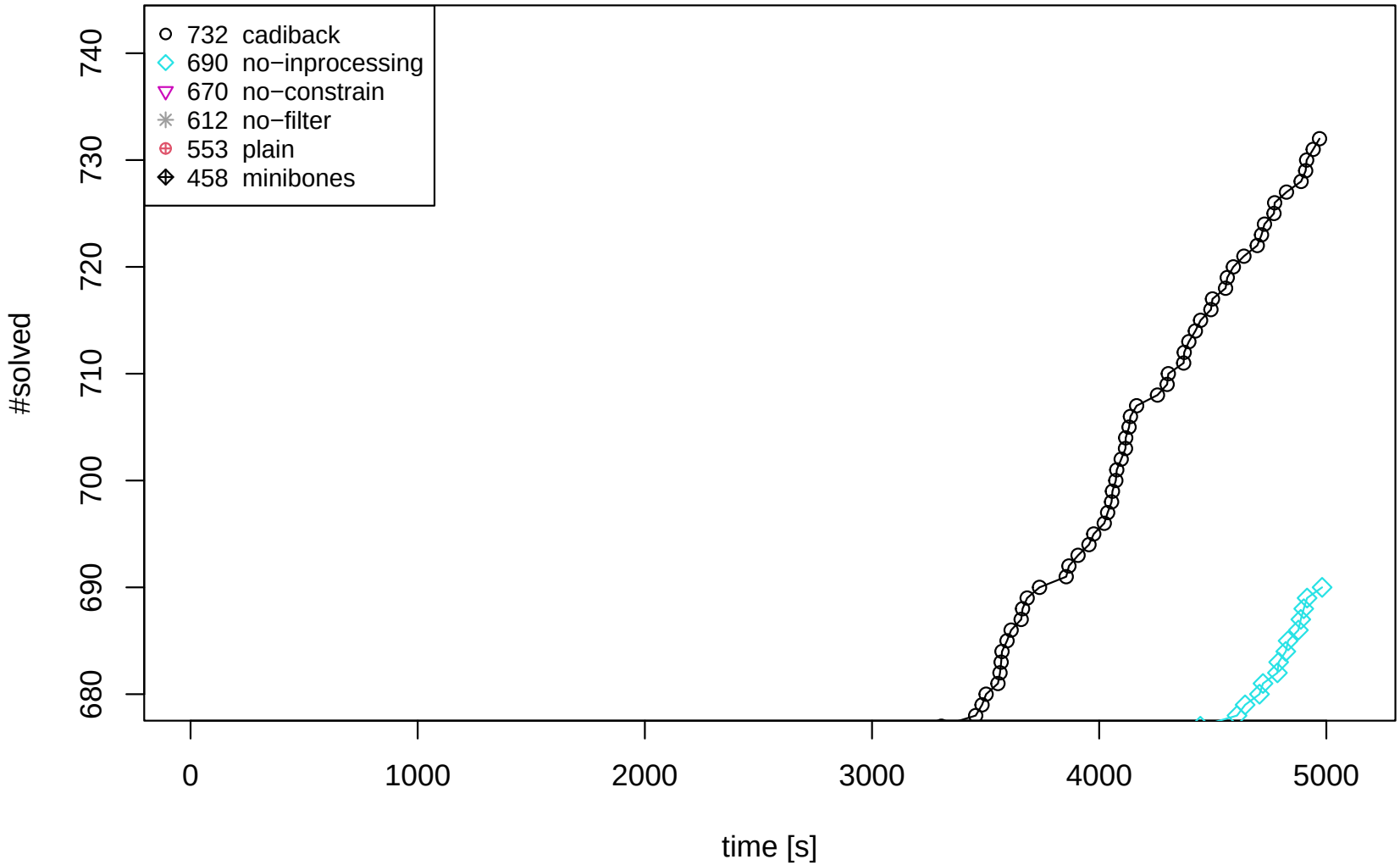
458 minibones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Flippable Literals via Watch Lists

Literal ℓ is flippable in model σ `iff` $\sigma \setminus \{\ell\} \cup \{\neg\ell\}$ is a model

flippable (CNF φ, literal ℓ, model σ)

1 **for** all clauses C watched by ℓ in φ

2 **if** $\sigma(C \setminus \{\ell\}) = 0$ **then return** 0

3 **return** 1

Kissat [Fleury and Biere, 2022]

Backbone Extraction

backbone (CNF φ)

1 $(res, \sigma) \leftarrow \mathsf{SAT}(\varphi)$

2 $\mathcal{B} \leftarrow \emptyset, \quad \Lambda \leftarrow \sigma\{\ell \in \sigma \mid \neg\mathsf{flippable}(\ell, \sigma)\}, \quad k \leftarrow 1$

3 **while** $\Lambda \neq \emptyset$ **do**

4 $\quad \Gamma \leftarrow$ pick k literals from Λ // chunk

5 $\quad (res, \sigma) \leftarrow \mathsf{SAT}(\varphi \mid \bigvee_{\ell \in \Gamma} \neg\ell)$

6 $\quad$ **if** res **then** // satisfiable

7 $\quad\quad \Lambda \leftarrow \Lambda \cap \sigma\{\ell \in \sigma \mid \neg\mathsf{flippable}(\ell, \sigma)\}, \quad k \leftarrow 1$

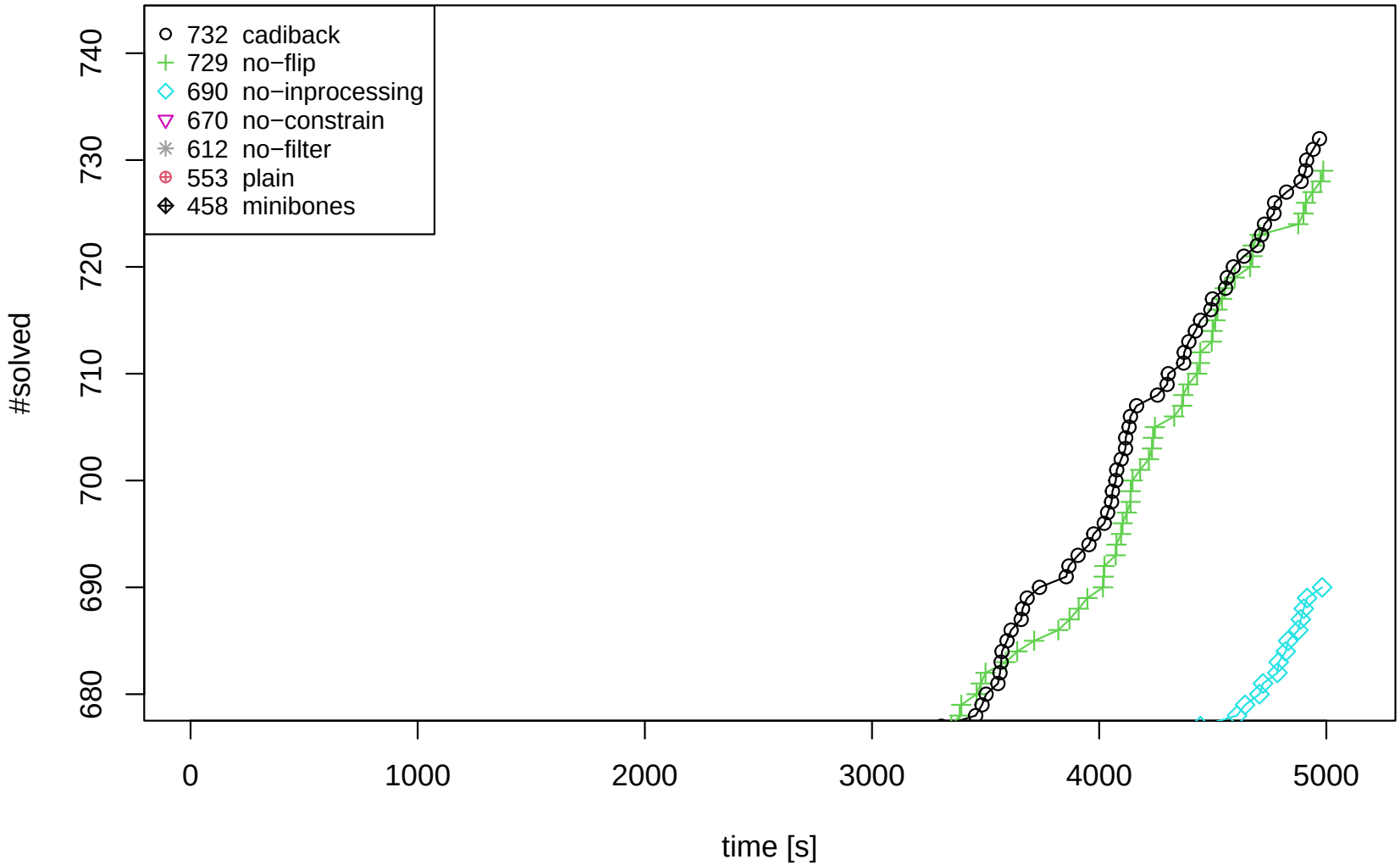
8 $\quad$ **else** // unsatisfiable

9 $\quad\quad \mathcal{B} \leftarrow \mathcal{B} \cup \Gamma$

10 $\quad\quad \Lambda \leftarrow \Lambda \setminus \Gamma$

11 $\quad\quad k \leftarrow |\Lambda|$

12 **return** $\mathcal{B}$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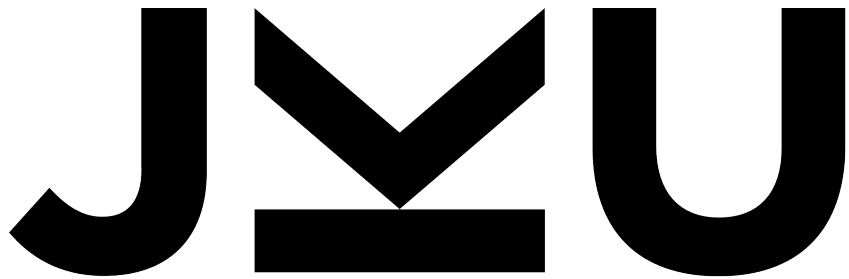
Antithesis 3: Flippable literals do not work

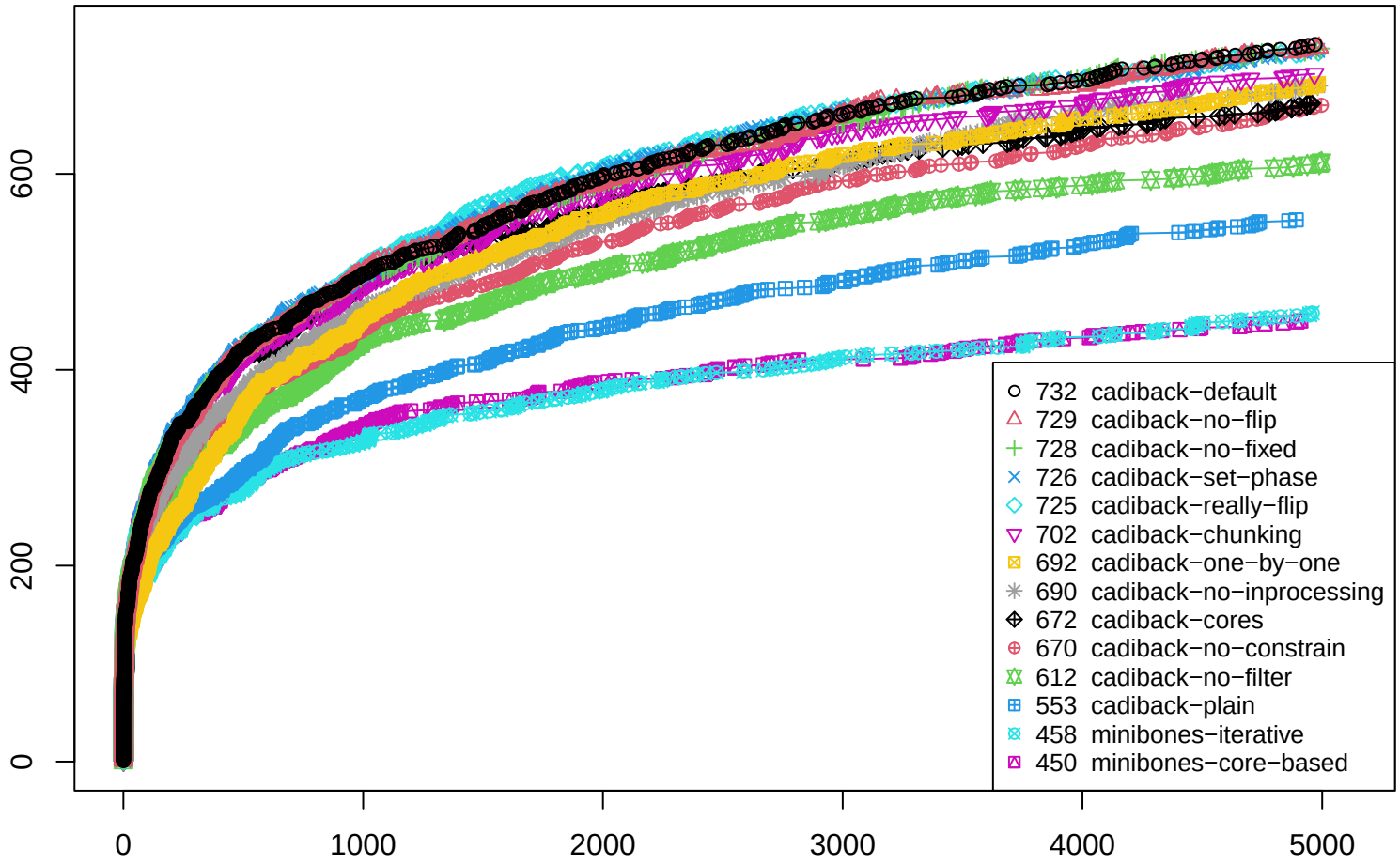
"However, the rotatable [flippable] literals technique yields generally a slower algorithm than the one without it."

Algorithms for Computing Backbones of Propositional Formulae
– Mikolás Janota, Inês Lynce, João Marques-Silva
[Janota et al., 2015]



JOHANNES KEPLER
UNIVERSITY LINZ

Full CDF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725 cadiback−really−flip
702 cadiback−chunking
692 cadiback−one−by−one
690 cadiback−no−inprocessing
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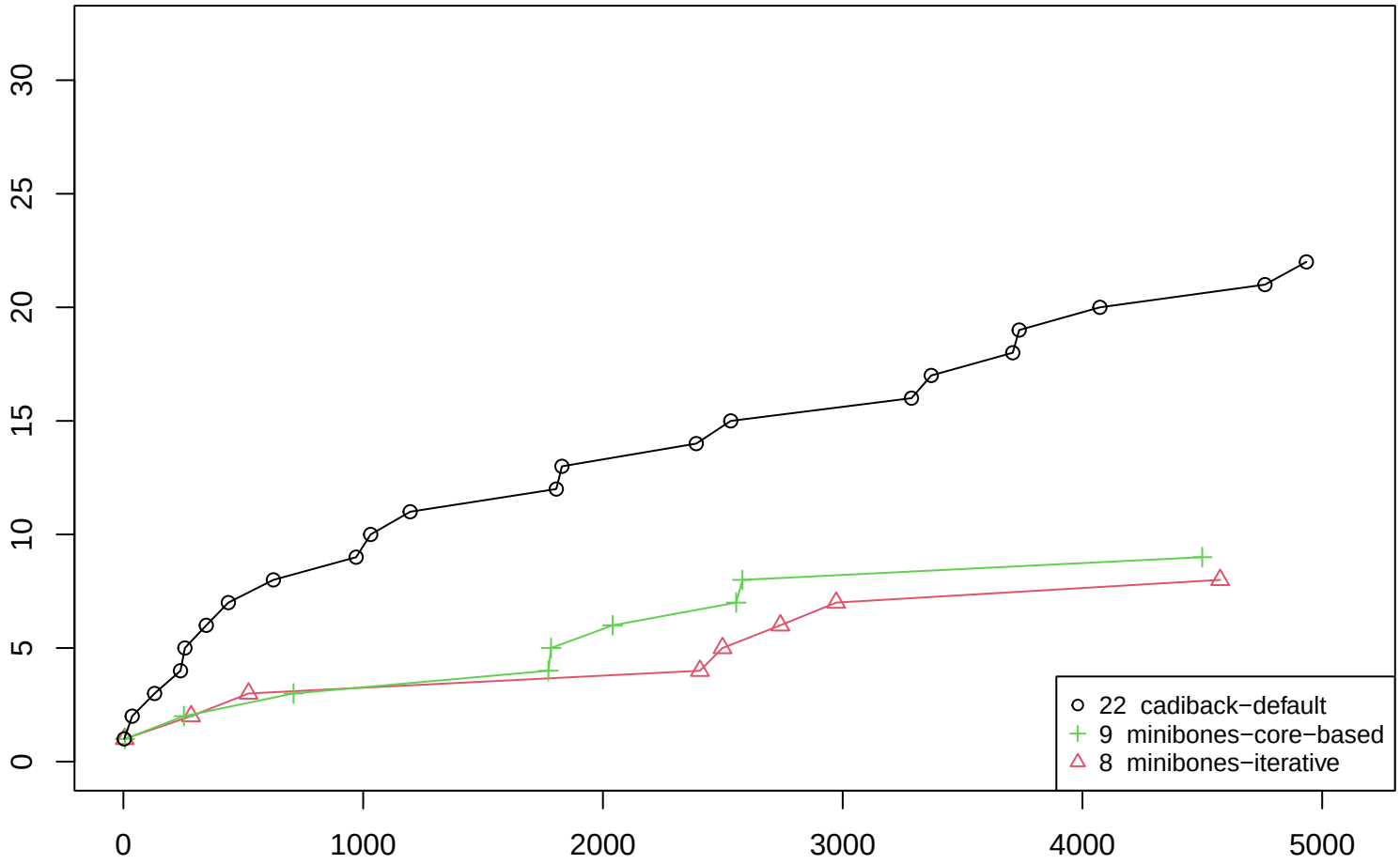
670 cadiback−no−constrain
612 cadiback−no−filter
553 cadiback−plain
458 minibones−iterative
450 minibones−core−based

SC2020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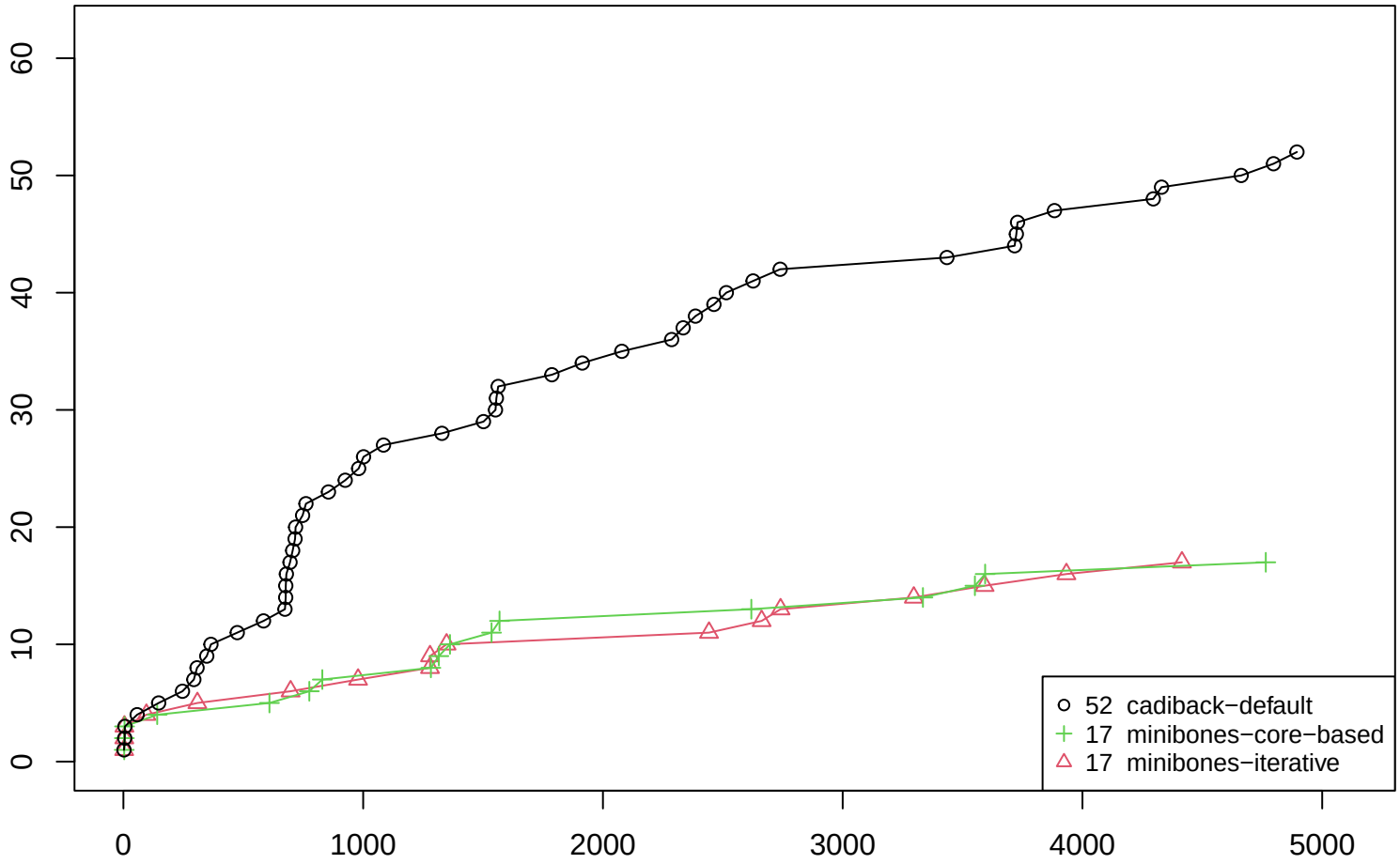
SC2021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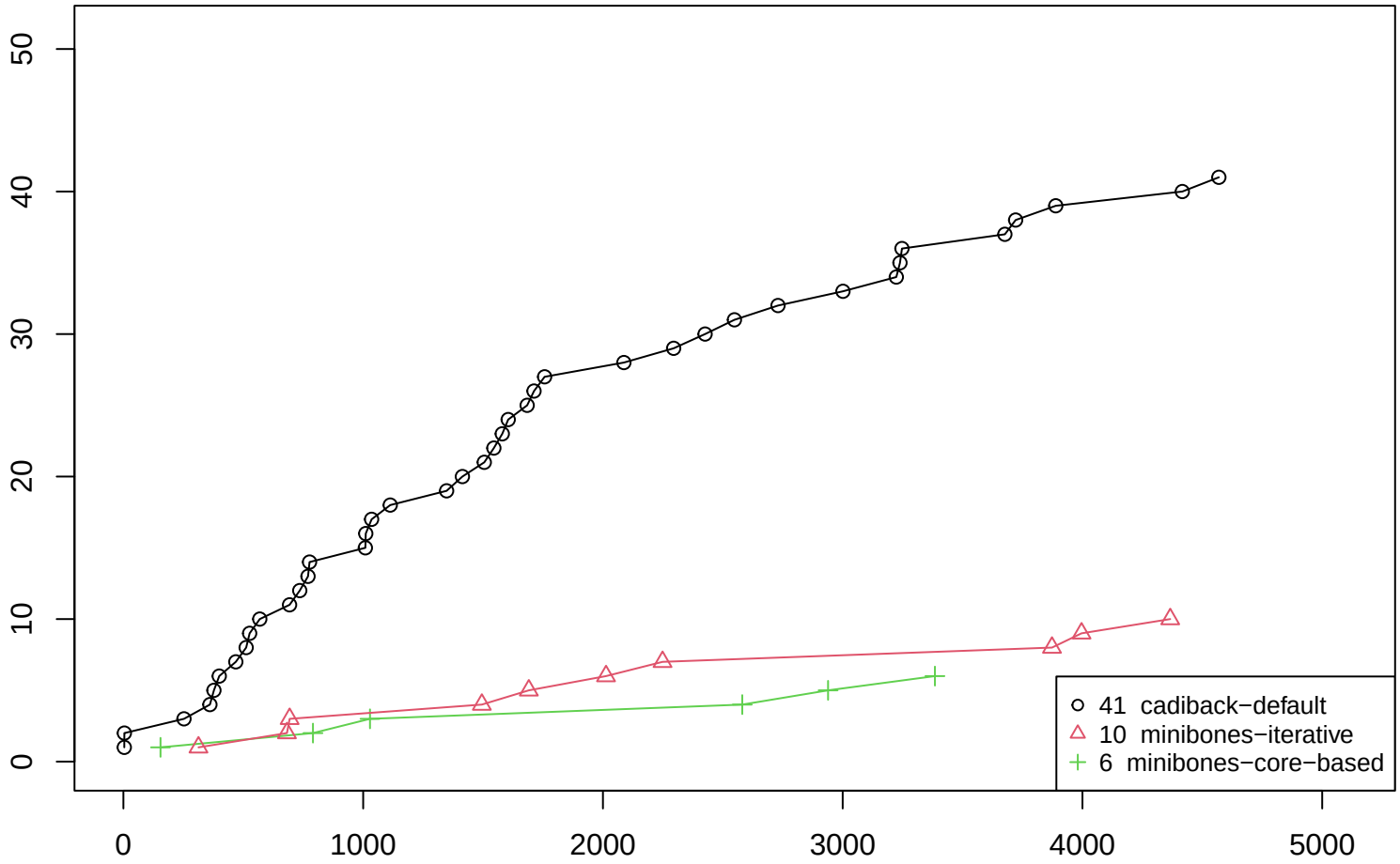
SC2022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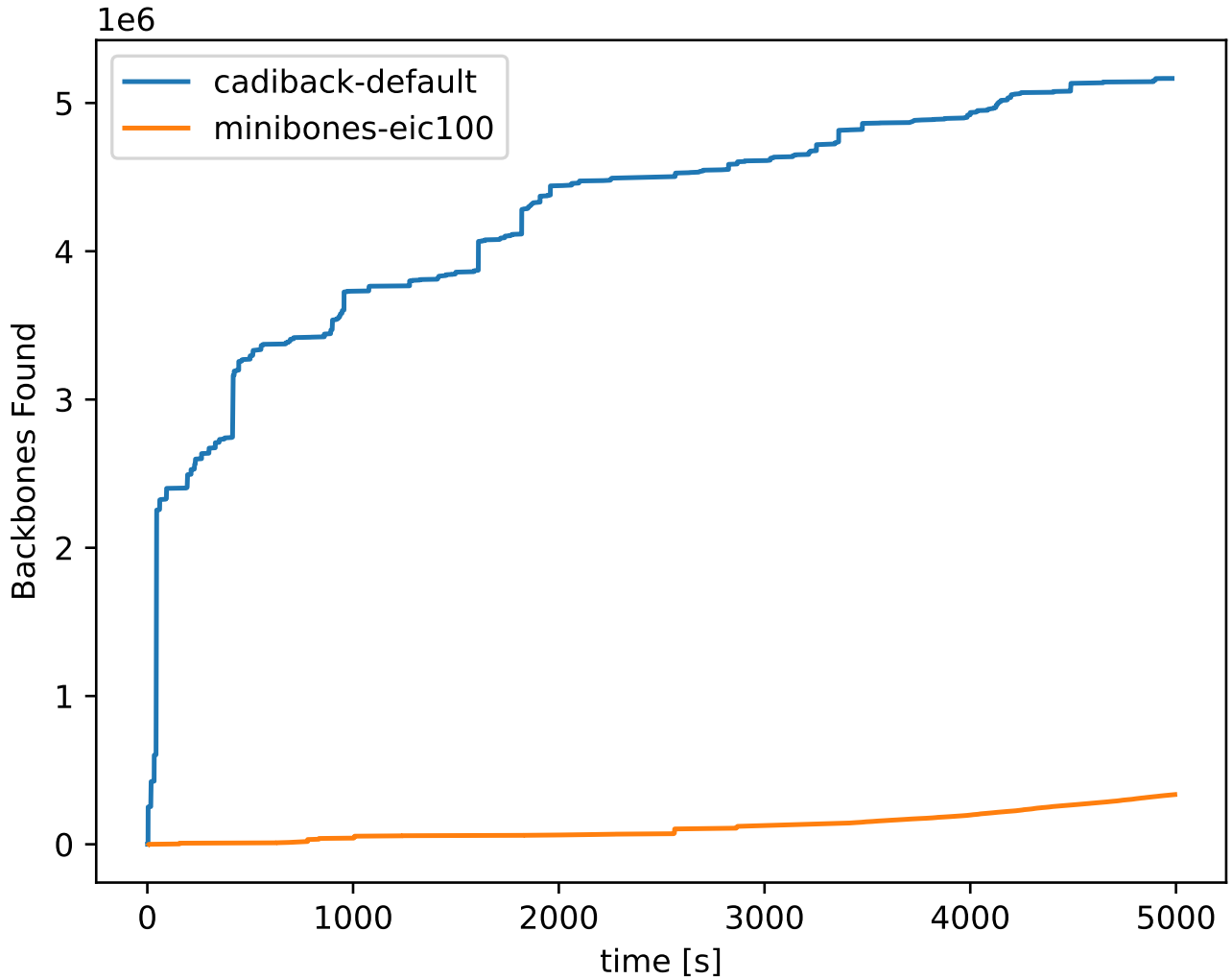
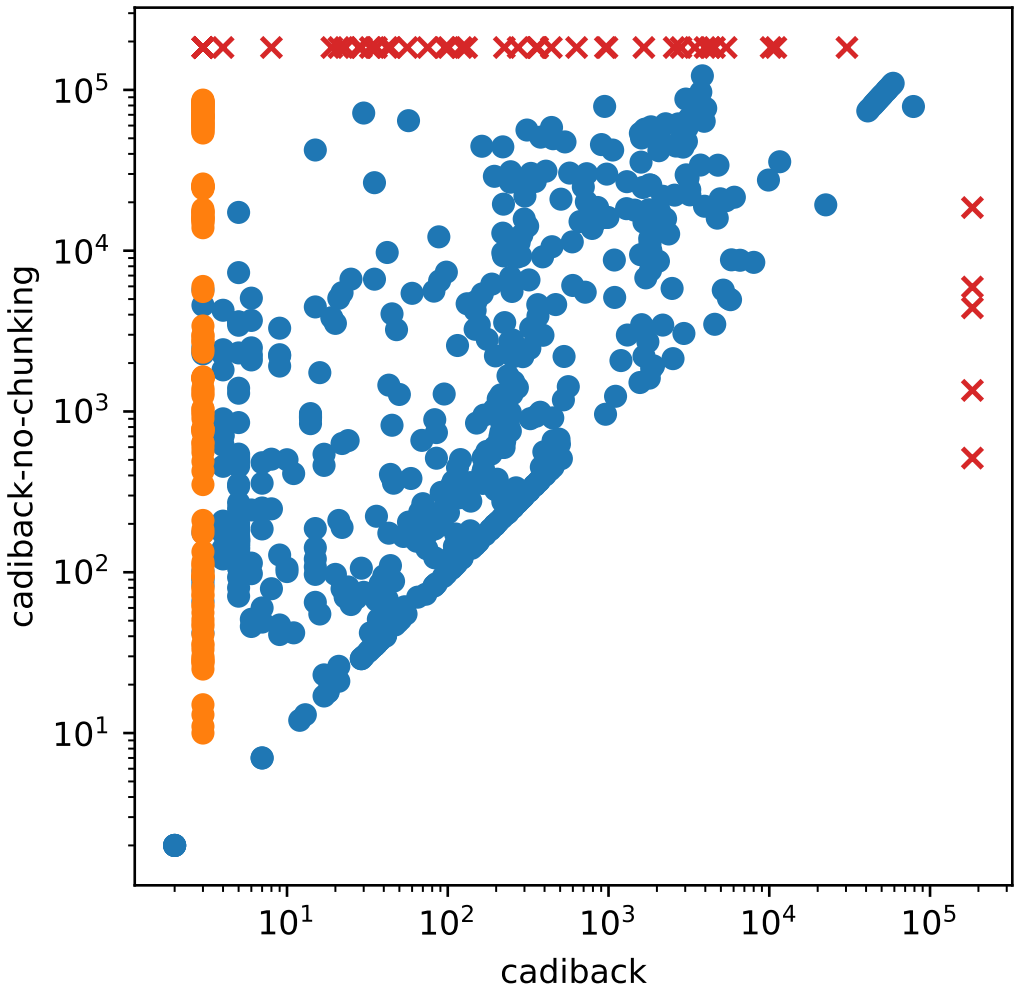
Identified Backbones in SC2022



Number of SAT Calls



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