LS-DTKMS: A Local Search Algorithm for Diversified Top-k MaxSAT Problem

Junping Zhou[1], **Jiaxin Liang**[1], Minghao Yin[1,2], Bo He[1]

[1]School of Information Science and Technology
[2]Key Laboratory of Applied Statistics of MOE
Northeast Normal University, China
{*zhoujp877, liangjx348, ymh, heb020*}*@mail.com*

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1. Background
2. Algorithm
3. Experiments
4. Future Work

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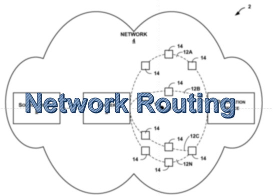
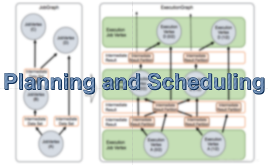
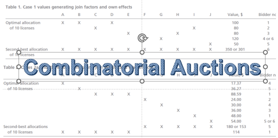
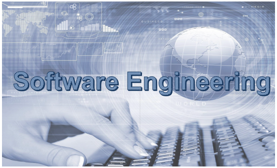
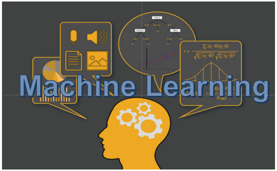
The success of the MaxSAT algorithms has contributed to a significant number of applications











The importance of diverse solutions in many applications

- **Social network:** diverse communities

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- **Social network:** diverse communities
- **Robotic motion planning:** diverse paths
- **Natural language understanding:** diverse plausible parses of a sentence
- **Computational biology:** diverse configurations of a protein structure

Definition of DTKMS

Diversified Top-k MaxSAT (**DTKMS**) Problem

- Given a formula $F = \mathit{Hard} \cup \mathit{Soft}$ and a positive integer k
- **Diversified Top-k MaxSAT problem** returns a set S with at most k feasible assignments
  - **Maxmize** $|sat(\alpha_1) \cup sat(\alpha_2) \cup \cdots \cup sat(\alpha_k)|$
  - $\alpha_i \in S$ is a feasible assignment, $sat(\alpha_i)$ is the set of soft clauses satisfied by α_i
- Equivalent to MaxSAT when $k = 1$

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Advantage

- DTKMS is a bi-standard optimization problem that balances correlation and diversity
- Provide a better result for users to better meet their needs

Straightforward method

- The method involves first exhaustively searching for all feasible assignments and then employing the max k-coverage algorithm
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Challenges

- Avoid generating highly overlapping solutions
- Without an exhaustive search, how can the quality of the solutions be guaranteed

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Overview of LS-DTKMS

We propose a local search algorithm for DTKMS, named LS-DTKMS.

Main idea

- **Construct the candidate solution set storing k feasible assignments**
  - Find feasible assignments
    - How?
- **Update the top-k candidate solution set**
  - Replace old assignment with new assignment
    - Which?
    - How?

Overview of LS-DTKMS

Algorithm 1: LS-DTKMS

Input: a DTKMS instance $F = Hard \cup Soft$, an integer k, and a *cutoff* time

Output: a solution of DTKMS S^*

$m = m_0$, $S^* = \emptyset$;
while *elapsed time* $<$ *cutoff* **do**
    `// ` m_{max} `and` m_0 `are parameters`
    if $m < m_{max}$ **then** $m \leftarrow 2 \times m$;
    else $m_0 \leftarrow m_0 + 1$, $m \leftarrow m_0$;
    for *each soft clause* c **do**
        $ClauseConf[c] = 1$
    $S \leftarrow$ ConstructS(F, m);
    if $|sat(S)| = |Soft|$ **then return** S;
    $S \leftarrow$ UpdateS(F, S, m);
    if $|sat(S)| > |sat(S^*)|$ **then** $S^* \leftarrow S$;
return S^*;

- m is a parameter used in the Best From Multiple Selections (BMS) strategy [Cai 2015]
- $ClauseConf$ is a Boolean array to express the state of each soft clause

Finding a Feasible Assignment with Diversity

Score of Variable

Given a DTKMS formula F, a candidate solution set S containing k assignments, and an assignment $\alpha \in S$, the score of a variable v is defined as:

$$score(v) = \lambda_1 \cdot score_1(v) + \lambda_2 \cdot score_2(v)$$

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$$score(v) = \lambda_1 \cdot score_1(v) + \lambda_2 \cdot score_2(v)$$

- Impact on the quality of the candidate solution set

$$score_1(v) = |sat((S \setminus \{\alpha\}) \cup \{\alpha'\})| - |sat(S)|$$

- Impact on the current assignment

$$score_2(v) = make(v) - break(v)$$

Finding a Feasible Assignment with Diversity

Variable Selection Heuristic

(1) **Initial assignment construction phase**

With a certain probability, the initial assignment is generated randomly or generated from the previous feasible assignment

- $ClauseConf$: $ClauseConf[c]{=}1$ identifies the soft clause c is falsified and unselected during each loop iteration in LS-DTKMS; otherwise, $ClauseConf[c]{=}0$
- prefer to choose a flipping variable in c with $ClauseConf[c]{=}1$ to satisfy as many soft clauses in $Soft \setminus sat(S)$ as possible
- $ClauseConf$ is initialized to 1. $ClauseConf[c]$ is updated as the soft clause is selected and the satisfaction state changes
- Select flipping variable in:
  - select a random falsified soft clause under α in $C = \{c | \text{ClauseConf}[c] = 1\}$
  - select a random falsified soft clause in $Soft \setminus sat(S)$

Finding a Feasible Assignment with Diversity

Variable Selection Heuristic

(2) **local search phase**

two-priority-level heuristic

- The first priority level:
  If there exist variables in $D = \{v|score(v) > 0\}$, choose a variable with BMS strategy, breaking ties by selecting the one that is least recently flipped.
- The second priority level:
  $D = \{v|score(v) > 0\} \neq \emptyset$. update the weights of the clauses [Lei and Cai, 2018]
  - select a random falsified hard clause c and a variable in c
  - select a random falsified soft clause c and a variable in c

Updating the Solution Set

Score of Assignment

Given a set of feasible assignments, $S = \{\alpha_1, \alpha_2, \cdots, \alpha_k\}$, the private satisfied soft clauses of α_i ($\alpha_i \in S$), denoted by $priv(\alpha_i, S)$, is the subset of soft clauses only satisfied by α_i, i.e.,

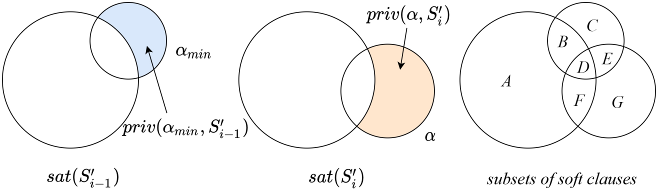
$$priv(\alpha_i, S) = sat(\alpha_i) \setminus sat(S \setminus \{\alpha_i\})$$

The score of α_i is defined as:

$$score(\alpha_i) = |priv(\alpha_i, S)|$$

- Which one to be replaced
- What is the replacement condition

Updating the Solution Set



- Which one to be replaced
  - score of assignment (the private soft clauses)
  - α_{min}: assignment with the lowest $score(\alpha)$
- What is the replacement condition

$$|priv(\alpha, S')| > |priv(\alpha_{min}, S)| + \frac{|sat(S)|}{|S|}$$

Advantages of LS-DTKMS

Diverse feasible assignments

- Randomness in generating assignments
- Flexible variable scoring function that takes into account the gain of flipping variables from both local and global perspectives.
- Two variable selection heuristics for diversity

Guaranteed solution quality

- Achieve the approximation ratio of 0.25, and much better in practice

Low memory

- Maintain the solution set with only k assignments

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Settings

Benchmarks:

- **Top-k MaxSAT instances:** from MaxSAT Evaluation 2020 top-k track
- **Diversified Top-k Clique Search (DTKCS) problem instances:** from Second DIMACS Implementation Challenge, and modeled into DTKMS instances by usual encoding

Competitors:

- **Top-k MaxSAT solvers:**
  MaxHS, Open-WBO, maxino, and RC2-A
- **DTKCS incomplete solvers:**
  TOPKLS and HEA-D

Time limit: 600s
Number of runs of an incomplete solver: 10

Comparison of LS-DTKMS and Top-k MaxSAT solvers

Table 1: Description of the top-k track benchmarks (29 families).

Families	$\|Var\|$	$\|Hard\|$	$\|Soft\|$
aes (1)	147	240	147
aes-key-recovery (1)	21368	372240	407
atcoss (2)	192216	890542	301
bcp (2)	152	462	125
CircuitDebuggingProblems (2)	144580	0	432737
CircuitTraceCompaction (2)	158890	494821	20
close_solutions (4)	68053	2600438	67868
ConsistentQueryAnswering (3)	44526	46593	10856
des (3)	86878	388071	4707
drmx-atmostk (3)	927	1408	47
fault-diagnosis (1)	137900	758138	49770
frb (1)	760	41367	760
gen-hyper-tw (1)	62531	197071	48
maxclique (3)	182	16885	182
MaximumCommonSub-GraphExtraction (3)	2044	99615	41

Families	$\|Var\|$	$\|Hard\|$	$\|Soft\|$
maxone (2)	485	2817	485
MaxSATQueriesinInter-pretableClassifiers (5)	1991	14448	1058
mbd (2)	29703	73188	4637
packup (5)	13138	73769	7572
protein_ins (5)	2253	2190700	59
pseudoBoolean (3)	839	1610	815
railway-transport (2)	64743	1668566	4945
ramsey (1)	36	0	210
reversi (6)	2981	16521	45
scheduling (1)	201204	767081	1707
SeanSafarpour (1)	238290	0	936006
treewidth-computation (3)	88809	814018	60
uaq (3)	2205	3805	189
xai-mindset2 (1)	1330	4763	378

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Comparison of LS-DTKMS and Top-k MaxSAT solvers

Families	k=2					k=3				
	Open-WBO	MaxHS	Maxino	RC2-A	LS-DTKMS	Open-WBO	MaxHS	Maxino	RC2-A	LS-DTKMS
aes (1)	NA(600.00)	129(25.34)	130(**0.23**)	130(0.20)	**147**(7.38)	NA(600.00)	132(25.34)	132(**0.23**)	132(0.19)	**147**(8.37)
aes-key-recovery (1)	NA(600.00)	406(600.00)	NA(600.00)	406(600.00)	**407**(**40.92**)	NA(600.00)	406(600.00)	NA(600.00)	406(600.00)	**407**(**62.34**)
atcoss (2)	265.5(364.80)	265.5(**23.54**)	265.5(25.87)	265.5(43.51)	**290.5**(53.49)	265.5(102.71)	265.5(**23.96**)	265.5(26.24)	265.5(43.83)	**290.5**(56.31)
bcp (2)	73.5(0.02)	73.5(0.01)	73.5(0.01)	73.5(**0.00**)	**74**(7.43)	73.5(0.02)	73.5(**0.01**)	73(**0.01**)	73.5(**0.01**)	**74**(7.93)
CircuitDebugging-Problems (2)	432736 (322.08)	432736 (299.53)	432736 (**4.09**)	432736 (4.31)	**432737** (23.71)	432736 (322.25)	**432736.5** (299.76)	432736 (4.22)	432736 (**6.41**)	**432736.5** (24.69)
CircuitTraceCo-mpaction (2)	12.5(22.60)	12(37.54)	12.5(**9.52**)	12.5(20.99)	**14**(13.61)	13(22.02)	13(38.90)	13(9.58)	13(20.28)	**16.5**(**11.55**)
close_solutions (4)	67857 (44.56)	67857 (178.48)	67857.3 (12.45)	67857 (**2.45**)	**67861.3** (28.55)	67858.3 (43.71)	67858.3 (158.82)	67858.3 (12.46)	67858.3 (**2.92**)	**67862** (31.66)
ConsistentQuery-Answering (3)	**0**(1.63)	**0**(**0.05**)	**0**(0.25)	**0**(0.05)	**0**(21.42)	**0**(1.62)	**0**(**0.06**)	**0**(0.25)	**0**(**0.06**)	**0**(23.06)
des (3)	3978(251.09)	3977(333.09)	4699.3(112.24)	4700(**30.56**)	**4701.7**(103.27)	3978(251.61)	3978(251.17)	4699.3(112.79)	4700(**30.25**)	**4701.7**(61.81)
drmx-atmostk (3)	29(0.67)	29(98.53)	19(201.58)	28.7(21.05)	**32**(132.99)	29.3(**0.68**)	30(94.93)	19(201.60)	29.7(21.33)	**44**(125.54)
fault-diagnosis (1)	**49593** (34.63)	**49593** (122.39)	**49593** (24.46)	**49593** (**7.01**)	**49593** (8.22)	**49593** (38.30)	**49593** (150.28)	**49593** (25.17)	**49593** (**7.39**)	**49593** (8.43)
frb (1)	40(46.34)	NA(600.00)	NA(600.00)	42(**6.97**)	**54**(84.00)	40(46.56)	NA(600.00)	NA(600.00)	43(**7.14**)	**77**(89.27)
gen-hyper-tw (1)	**44**(128.45)	**44**(**117.65**)	**44**(233.96)	**44**(117.94)	**44**(121.98)	**44**(131.84)	**44**(61.67)	**44**(242.70)	**44**(**116.78**)	**44**(127.47)
maxclique (3)	18.7(1.37)	20(56.54)	19.7(4.91)	19.3(**0.56**)	**30**(11.58)	116.3(1.39)	116(61.49)	116.3(4.96)	116.3(**0.00**)	**135.7**(10.55)
MaximumCom-monSub-Graph-Extraction (3)	19(226.61)	20.3(130.08)	20.3(122.02)	20(**107.98**)	**22.3**(149.94)	20(224.80)	21.3(218.50)	21.3(120.73)	21(**105.84**)	**25**(152.61)
maxone (2)	238.5(0.18)	238(0.18)	238.5(**0.05**)	238(0.21)	**254**(79.91)	245(0.18)	240(0.18)	240(**0.05**)	238(0.23)	**257.5**(73.90)
MaxSATQuerie-sinInterpretable-Classifiers (5)	527.2(23.11)	553.3(89.70)	525.6(**1.98**)	526.6(6.21)	**556**(68.68)	528.2(23.48)	554.5(120.08)	526.6(**1.98**)	528.2(6.62)	**557.2**(69.06)
mbd (2)	4616(198.04)	4616(61.28)	4615.5(1.70)	4616(0.59)	**4636**(**0.17**)	4617(194.40)	4617(58.63)	4616.5(1.70)	4617(0.61)	**4636**(**0.19**)
packup (5)	6959.8(11.81)	6957.4(6.58)	6958.2(1.03)	6959.2(**0.78**)	**7209.8**(25.10)	6965.4(11.98)	6966.6(6.88)	6966(1.03)	6964.8(**0.91**)	**7209.8**(29.17)
protein_ins (5)	29.6(85.07)	30.4(98.79)	30(206.86)	29.6(**33.40**)	**38.2**(140.30)	30.6(88.13)	31(192.14)	30.6(188.86)	29.8(**33.89**)	**39.8**(104.50)
pseudoBoolean (3)	408(0.81)	408(0.05)	407.7(0.07)	408(**0.04**)	**409**(1.70)	408(0.78)	408.3(0.05)	407.7(0.07)	408(**0.05**)	**410**(2.23)
railway-transport (2)	4920.5(27.41)	4921(44.00)	4920.5(6.67)	4920.5(**4.36**)	**4932.5**(14.94)	4775.5(27.95)	4775.5(34.61)	4775.5(6.71)	4775.5(**4.43**)	**4932.5**(19.35)
ramsey (1)	**210**(**0.10**)	**210**(0.65)	**210**(0.72)	**210**(0.31)	**210**(16.27)	**210**(**0.10**)	**210**(0.66)	**210**(0.69)	**210**(0.32)	**210**(16.32)
reversi (6)	32.3(**2.01**)	31.8(10.50)	31.8(6.52)	32.17(4.97)	**37.3**(46.55)	33(**2.08**)	32.7(9.97)	32.5(6.55)	32.8(5.26)	**39.3**(47.57)
scheduling (1)	1478(600.00)	1476(209.98)	1478(**29.94**)	1478(107.71)	**1481**(297.34)	1478(600.00)	1477(600.00)	1478(**29.94**)	1478(104.15)	**1481**(162.31)
SeanSafarpour (1)	NA(600.00)	NA(600.00)	**936004** (312.62)	**936004** (**44.42**)	NA(600.00)	NA(600.00)	NA(600.00)	**936004** (320.09)	**936004** (**48.05**)	NA(600.00)
treewidth-compu-tation (3)	**52.8**(28.90)	**52.8**(26.56)	**52.8**(16.11)	**52.8**(**15.32**)	**52.8**(17.99)	**52.8**(37.75)	**52.8**(33.12)	**52.8**(16.09)	**52.8**(**15.25**)	**52.8**(20.22)
uaq (3)	140.3(10.06)	136.7(16.81)	137.3(25.72)	141.3(**7.88**)	**170**(79.88)	140.3(10.08)	138.3(17.95)	138.3(26.89)	149.3(**7.97**)	**173**(84.54)
xai-mindset2 (1)	374(0.07)	372(0.06)	374(**0.02**)	359(**0.02**)	**376**(22.43)	374(0.07)	372(0.07)	374(0.03)	360(**0.01**)	**376**(27.31)

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	k=4					k=5				
Families (29)	Open-WBO	MaxHS	Maxino	RC2-A	LS-DTKMS	Open-WBO	MaxHS	Maxino	RC2-A	LS-DTKMS
aes (1)	NA(600.00)	132(25.53)	132(0.23)	132(**0.20**)	**147**(6.12)	NA(600.00)	132(25.95)	132(0.23)	132(**0.21**)	**147**(5.12)
aes-key-recovery (1)	NA(600.00)	406(600.00)	NA(600.00)	406(600.00)	**407**(**73.34**)	NA(600.00)	406(600.00)	NA(600.00)	406(600.00)	**407**(**75.12**)
atcoss (2)	265.5(108.59)	265.5(25.35)	265.5(22.44)	265.5(**53.06**)	**290.5**(57.96)	265.5(110.47)	265.5(26.79)	265.5(**22.90**)	265.5(57.42)	**290.5**(62.12)
bcp (2)	73.5(0.01)	73.5(0.01)	73(0.01)	73.5(**0.00**)	**74**(7.78)	73.5(**0.01**)	73.5(**0.01**)	73(**0.01**)	73.5(**0.01**)	**74**(6.99)
CircuitDebugging-Problems (2)	432736 (321.45)	**432736.5** (299.95)	432736 (**5.33**)	432736 (9.11)	**432736.5** (23.08)	**432736.5** (322.19)	**432736.5** (300.17)	**432736.5** (**6.04**)	**432736.5** (10.00)	**432736.5** (24.57)
CircuitTraceCo-mpaction (2)	13(20.56)	13(26.65)	13(**6.31**)	13(26.10)	**18.5**(14.03)	13(21.41)	13(30.88)	13(**6.25**)	13(26.68)	**20**(14.28)
close_solutions (4)	67858.3 (45.49)	67858.3 (87.84)	67858.3 (17.21)	67858.3 (**4.35**)	**67866** (37.92)	67858.3 (44.75)	67858.3 (130.86)	67858.3 (17.03)	67858.3 (**4.81**)	**67866** (40.90)
ConsistentQuery-Answering (3)	**0**(1.63)	**0**(0.05)	**0**(0.24)	**0**(**0.04**)	**0**(21.38)	**0**(1.65)	**0**(**0.05**)	**0**(0.25)	**0**(**0.05**)	**0**(22.46)
des (3)	3978.5(246.31)	3978.5(329.68)	4700(122.37)	4700.7(**31.35**)	**4701.7**(58.56)	4706.5(244.65)	4706.5(213.85)	4625.3(110.95)	4626(**55.35**)	**4626.3**(113.76)
drmx-atmostk (3)	31(0.67)	31.7(99.74)	22(201.60)	31.3(**21.58**)	**46.7**(116.70)	31(0.66)	31.7(96.77)	22(201.63)	31.7(**21.65**)	**46.7**(116.83)
fault-diagnosis (1)	**49593** (30.24)	**49593** (150.27)	**49593** (20.74)	**49593** (**7.59**)	**49593** (11.28)	**49593** (30.27)	**49593** (180.01)	**49593** (20.25)	**49593** (**7.69**)	**49593** (8.12)
frb (1)	45(43.69)	NA(600.00)	NA(600.00)	45(**7.00**)	**110**(99.09)	45(44.15)	NA(600.00)	NA(600.00)	45(**7.04**)	**122**(123.14)
gen-hyper-tw (1)	**44**(178.53)	**44**(**105.23**)	**44**(221.92)	**44**(116.71)	38(88.79)	**44**(181.03)	**44**(188.43)	**44**(223.14)	**44**(112.24)	42(**94.13**)
maxclique (3)	117(2.03)	116.7(60.98)	117(5.03)	117(**0.01**)	**143**(11.09)	117(1.50)	116.7(138.22)	117(5.00)	117(**0.00**)	**150.3**(10.49)
MaximumCom-monSub-Graph-Extraction (3)	23(219.66)	22.3(187.72)	23(107.63)	23(**104.37**)	**26**(151.78)	23(224.80)	22.3(197.61)	23(107.63)	23.7(**105.67**)	**27**(149.65)
maxone (2)	245(0.23)	243(0.18)	245(**0.05**)	238(0.22)	**258**(69.85)	248(0.17)	245(0.19)	245.5(**0.03**)	238(0.23)	**258**(64.77)
MaxSATQuerie-sinInterpretable-Classifiers (5)	528.2(22.02)	554.5(120.58)	526.6(**1.84**)	528.2(6.87)	**557.2**(67.97)	528.2(23.08)	554.5(110.08)	526.6(**1.87**)	528.2(6.93)	**557.2**(64.76)
mbd (2)	4617(259.82)	4617(42.93)	4616.5(1.85)	4617.5(0.69)	**4636**(**0.29**)	4617.5(255.90)	4617(47.07)	4617(1.83)	4618(0.67)	**4636**(**0.16**)
packup (5)	6972.4(11.34)	6972.4(7.04)	6973.2(0.99)	6971.2(**0.89**)	**7209.8**(28.74)	6972.6(11.38)	6972.6(7.32)	6973.4(0.98)	6973.2(**0.92**)	**7209.8**(26.52)
protein_ins (5)	31.6(91.45)	31.8(180.38)	31.6(209.63)	30.4(**34.42**)	**48**(104.57)	33.6(102.35)	33.6(174.08)	33.8(167.46)	31.4(**35.70**)	**55.4**(101.52)
pseudoBoolean (3)	408(0.82)	408.3(**0.05**)	408.3(0.07)	408(**0.05**)	**410**(1.96)	408(0.83)	408.3(**0.05**)	408.3(0.06)	408(**0.05**)	**410**(0.91)
railway-transport (2)	4777(21.63)	4777(38.16)	4777(7.38)	4777(**4.30**)	**4932.5**(23.28)	4777(27.77)	4777(38.07)	4777(7.28)	4777(**4.79**)	**4932.5**(24.23)
ramsey (1)	**210**(0.10)	**210**(0.70)	**210**(0.72)	**210**(**0.30**)	**210**(14.12)	**210**(**0.10**)	**210**(0.81)	**210**(0.73)	**210**(0.32)	**210**(10.31)
reversi (6)	33.8(**2.08**)	33.3(9.71)	33.3(6.92)	33.8(4.65)	**40.7**(48.64)	33.8(**2.21**)	33.3(19.36)	33.3(6.98)	33.8(5.12)	**41.3**(42.49)
scheduling (1)	**1481**(600.00)	1479(600.00)	1481(**37.57**)	1479(98.38)	**1481**(183.23)	1705(18.41)	1705(416.33)	1705(37.95)	1705(29.76)	**1707**(**18.01**)
SeanSafarpour (1)	NA(600.00)	NA(600.00)	**936004** (315.25)	**936004** (**45.29**)	NA(600.00)	NA(600.00)	NA(600.00)	**936004** (335.92)	**936004** (**56.57**)	NA(600.00)
treewidth-compu-tation (4)	**52.8**(41.45)	**52.8**(32.04)	**52.8**(14.79)	**52.8**(**14.38**)	**52.8**(18.57)	**52.8**(45.45)	**52.8**(31.81)	**52.8**(**14.75**)	**52.8**(15.61)	**52.8**(26.67)
uaq (3)	140.3(15.97)	138.3(20.87)	138.3(29.22)	155(**10.00**)	**183**(82.52)	140.3(16.46)	138.3(27.23)	138.3(31.06)	155(**11.57**)	**183**(73.48)
xai-mindset2 (1)	374(0.07)	372(0.08)	374(0.03)	361(**0.00**)	**376**(29.21)	374(0.07)	372(0.11)	374(**0.03**)	362(**0.03**)	**376**(45.13)

Comparison of LS-DTKMS and Top-k MaxSAT solvers

Summary

Table 2: Summary of the comparison of Open-WBO, MaxHS, Maxino, RC2-A, and LS-DTKMS on Top-k MaxSAT instances.

Instances	k	Open-WBO #Better	Open-WBO #Equal	MaxHS #Better	MaxHS #Equal	Maxino #Better	Maxino #Equal	RC2-A #Better	RC2-A #Equal	LS-DTKMS #Better	LS-DTKMS #Equal
Top-k MaxSAT instances (29 families)	2	0	4	0	5	0	5	0	5	24	4
	3	0	4	0	5	0	5	0	5	22	5
	4	0	6	0	5	0	6	0	6	21	5
	5	0	6	0	5	0	6	0	6	22	5
Total		0	20	0	20	0	22	0	22	89	19

Comparison of LS-DTKMS and DTKCS solvers

Table 3: Comparison on DTKCS with k=5 and k=10.

			k=5						k=10					
			TOPKLS		HEA-D		LS-DTKMS		TOPKLS		HEA-D		LS-DTKMS	
Instances (21)	|*Hard*|	|*Soft*|	*best*(*avg*)	*time*	*best*(*avg*)	*time*	*best*(*avg*)	*time*	*best*(*avg*)	*time*	*best*(*avg*)	*time*	*best*(*avg*)	*time*
brock200_2	10024	200	**52**(**52**)	110.63	**52**(**52**)	151.98	**52**(**52**)	53.96	93(93)	126.26	91(90)	210.78	**99**(**99**)	211.32
brock200_4	6811	200	73(73)	378.36	75(74)	169.93	**77**(**75**)	128.59	121(121)	36.53	133(130)	471.57	**135**(**133**)	35.17
brock400_2	20014	400	110(110)	187.87	115(112)	293.57	**120**(**120**)	207.95	192(192)	25.51	191(190)	263.77	**229**(**228**)	339.11
brock400_4	20035	400	119(119)	159.26	122(118)	352.24	**129**(**129**)	265	192(192)	530.14	191(190)	196.11	**231**(**231**)	225.13
C1000.9	49421	1000	271(270)	292.07	264(259)	534.38	**321**(**319**)	281.39	464(464)	497.64	474(471)	219.83	**507**(**507**)	116.46
C125.9	787	125	101(101)	73.04	113(112)	6.90	**123**(**122**)	5.89	104(104)	0.03	**125**(**125**)	0.00	122(120)	9.62
C250.9	3141	250	155(155)	553.33	173(167)	154.54	**193**(**193**)	28.48	206(206)	406.89	**250**(**249**)	0.02	**250**(246)	2.32
C500.9	12418	500	212(212)	222.91	224(219)	372.79	**257**(**256**)	147.64	323(321)	285.76	372(369)	329.21	**443**(**441**)	284.39
gen200_p0.9_44	1990	200	130(130)	561.09	149(145)	55.35	**174**(**174**)	223.79	166(166)	249.74	**200**(**200**)	0.00	**200**(197)	15.34
gen200_p0.9_55	1990	200	145(145)	392.94	160(157)	5.93	**185**(**185**)	106.08	174(174)	84.37	**200**(**200**)	0.00	**200**(**200**)	70.21
gen400_p0.9_55	7980	400	184(184)	193.76	200(197)	296.86	**219**(**207**)	209.95	264(264)	127.53	334(329)	179.58	**380**(**380**)	351.47
gen400_p0.9_65	7980	400	189(189)	283.49	217(210)	283.67	**250**(**250**)	218.55	274(274)	453.39	340(338)	262.97	**377**(**377**)	191.76
gen400_p0.9_75	7980	400	196(196)	402.29	240(232)	192.26	**265**(**265**)	220.12	276(276)	74.51	344(342)	113.96	**367**(**367**)	57.04
hamming8-4	11776	256	**80**(**80**)	10.05	**80**(**80**)	4.54	**80**(**80**)	8.33	**160**(**160**)	0.98	**160**(**160**)	75.85	**160**(**160**)	8.2
keller4	5100	171	**55**(**55**)	7.29	**55**(**55**)	11.01	**55**(**55**)	2.24	99(99)	274.42	98(97)	248.39	**110**(**110**)	114.16
MANN_a27	702	378	366(366)	524.8	**378**(**378**)	0.00	**378**(375)	260.1	**378**(**378**)	100.54	**378**(**378**)	0.00	**378**(**378**)	52.15
MANN_a45	1980	1035	973(973)	462.08	**1035**(**1035**)	0.01	**1035**(1032)	35.85	1034(1034)	62.14	**1035**(**1035**)	0.01	**1035**(**1035**)	66.36
MANN_a81	6480	3321	3036(3032)	335.67	**3321**(**3321**)	0.12	**3321**(3320)	31.47	3301(3301)	88.8	**3321**(**3321**)	0.12	**3321**(**3321**)	1.52
p_hat300-1	33917	300	**39**(**39**)	7.67	**39**(**39**)	55.04	**39**(**39**)	7.05	73(**73**)	85.72	68(68)	226.72	**74**(**73**)	268.54
p_hat300-2	22922	300	79(79)	222.49	80(77)	111.64	**94**(**93**)	55.15	110(110)	73.2	130(127)	373.91	**150**(**150**)	343.28
p_hat300-3	11460	300	108(108)	9.00	116(113)	232.32	**138**(**138**)	6.69	148(147)	251.25	193(187)	408.68	**195**(**195**)	109.7

Comparison of LS-DTKMS and DTKCS solvers

Table 4: Comparison on DTKCS instances with k=15 and k=20.

			k=15						k=20					
			TOPKLS		HEA-D		LS-DTKMS		TOPKLS		HEA-D		LS-DTKMS	
Instances (21)	|Hard|	|Soft|	*best*(*avg*)	*time*	*best*(*avg*)	*time*	*best*(*avg*)	*time*	*best*(*avg*)	*time*	*best*(*avg*)	*time*	*best*(*avg*)	*time*
brock200_2	10024	200	125(125)	296.52	128(126)	230.60	**143**(**143**)	70.22	146(146)	296.52	155(154)	372.79	**164**(**160**)	339.04
brock200_4	6811	200	151(151)	20.98	167(163)	410.85	**191**(**190**)	25.73	170(170)	20.98	192(188)	94.76	**198**(**196**)	44.24
brock400_2	20014	400	248(248)	491.45	264(263)	231.79	**320**(**320**)	220.73	292(292)	491.45	319(318)	240.85	**369**(**369**)	259.81
brock400_4	20035	400	252(252)	142.27	263(262)	305.84	**327**(**327**)	256.96	287(287)	142.27	325(318)	184.80	**360**(**360**)	180.30
C1000.9	49421	1000	587(587)	177.08	664(657)	136.11	**770**(**770**)	316.18	689(689)	177.08	841(833)	56.72	**823**(**823**)	372.70
C125.9	787	125	**125**(**125**)	0.00	**125**(**125**)	0.00	**125**(**125**)	19.74	**125**(**125**)	0.00	**125**(**125**)	0.00	**125**(**125**)	11.92
C250.9	3141	250	226(226)	86.74	**250**(**250**)	0.00	**250**(**250**)	119.19	223(223)	86.74	**250**(**250**)	0.00	**250**(**250**)	4.16
C500.9	12418	500	383(383)	57.54	490(488)	27.51	**500**(**500**)	142.67	418(418)	57.54	**500**(**500**)	0.00	**500**(**500**)	64.40
gen200_p0.9_44	1990	200	168(168)	2.83	**200**(**200**)	0.00	**200**(**200**)	9.16	**200**(**200**)	2.83	**200**(**200**)	0.00	**200**(**200**)	18.28
gen200_p0.9_55	1990	200	172(172)	1.61	**200**(**200**)	0.00	**200**(**200**)	11.77	**200**(**200**)	1.61	**200**(**200**)	0.00	**200**(**200**)	32.98
gen400_p0.9_55	7980	400	310(310)	43.22	**400**(**400**)	0.03	**400**(**400**)	9.42	326(325)	43.22	**400**(**400**)	0.01	**400**(**400**)	44.03
gen400_p0.9_65	7980	400	319(319)	88.31	**400**(**400**)	0.02	**400**(**400**)	12.81	341(341)	88.31	**400**(**400**)	0.01	**400**(**400**)	39.76
gen400_p0.9_75	7980	400	311(311)	442.95	**400**(**400**)	0.01	**400**(**400**)	11.35	335(335)	442.95	**400**(**400**)	0.00	**400**(**400**)	69.26
hamming8-4	11776	256	224(224)	69.23	**240**(229)	416.12	**240**(**240**)	41.98	246(246)	69.23	222(220)	353.77	**251**(**251**)	318.61
keller4	5100	171	126(126)	535.79	131(130)	218.73	**149**(**149**)	206.85	142(142)	535.79	152(**151**)	171.54	**153**(**151**)	52.11
MANN_a27	702	378	**378**(**378**)	0.14	**378**(**378**)	0.00	**378**(**378**)	6.13	**378**(**378**)	0.00	**378**(**378**)	0.00	**378**(**378**)	4.64
MANN_a45	1980	1035	**1035**(1033)	1.55	**1035**(**1035**)	0.01	**1035**(**1035**)	13.31	**1035**(**1035**)	1.55	**1035**(**1035**)	0.01	**1035**(**1035**)	57.91
MANN_a81	6480	3321	**3321**(**3321**)	338.63	**3321**(**3321**)	0.13	**3321**(**3321**)	116.70	**3321**(**3321**)	338.63	**3321**(**3321**)	0.13	**3321**(**3321**)	44.86
p_hat300-1	33917	300	**108**(105)	360.05	98(97)	198.69	**108**(107)	251.24	125(125)	360.05	121(118)	229.45	**140**(**140**)	224.28
p_hat300-2	22922	300	134(134)	57.61	162(159)	344.25	**191**(**191**)	257.23	144(144)	57.61	187(185)	194.29	**220**(**219**)	44.66
p_hat300-3	11460	300	168(168)	54.77	240(234)	156.88	**276**(**276**)	233.52	177(177)	54.77	275(**274**)	207.40	**276**(272)	46.18

Comparison of LS-DTKMS and DTKCS solvers

Summary

Table 5: Summary of the comparison of TOPKLS, HEA-D, and LS-DTKMS on DTKCS instances.

		TOPKLS		HEA-D		LS-DTKMS	
Instances	k	#Better	#Equal	#Better	#Equal	#Better	#Equal
DTKCS instances (21/37)	5	0	4	0	7	14	7
	10	0	2	1	7	13	7
	15	0	5	0	11	9	12
	20	0	6	0	11	10	11
Total		0	17	1	36	46	37

1. Background
2. Algorithm
3. Experiments
4. Future Work

Future Work

- Develop the algorithm through several strategies for diversity, e.g.:
  - Adjust the weighting scheme to highlight falsified soft clauses
  - Insert more randomness in the generation of a feasible assignment
- More applications for DTKMS
- Diversity enumeration for other problems

- For more details on the algorithm read our paper
- Download the instances
  - MSE top-k track: https://maxsat-evaluations.github.io/2020/benchmarks.html
  - DIMACS clique: https://iridia.ulb.ac.be/~fmascia/maximum_clique/
- For more data visit this repository https://github.com/LyreRabbit/DTKMS

Thank you!