Even shorter proofs without new variables

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Vienna University of Technology
Johannes Kepler University

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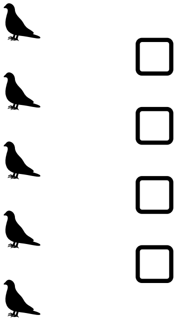
Problem 1: swapping pigeons

The pigeonhole problem $\mathrm{PHP}(n)$



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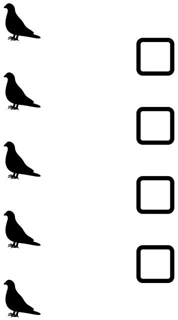

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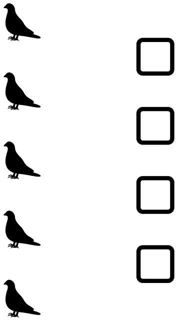

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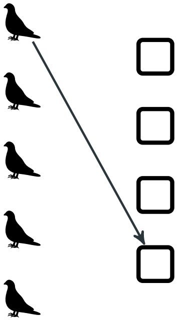

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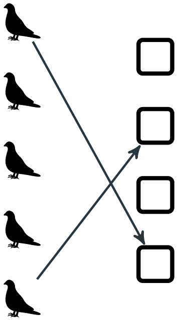

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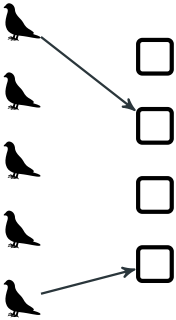

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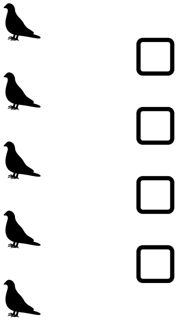

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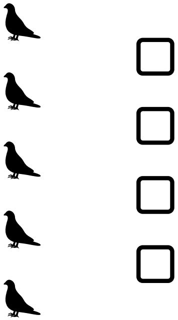

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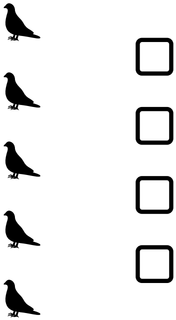

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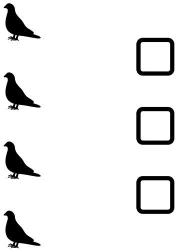

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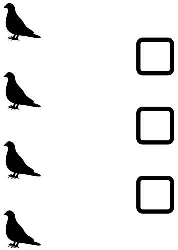

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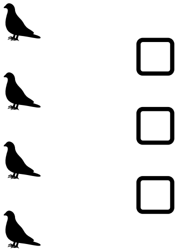

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How long are propositional proofs?

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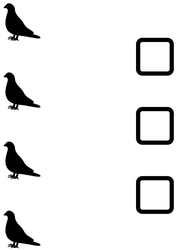

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Theoretical Computer Science 39 (1985) 297–308
North-Holland 297

THE INTRACTABILITY OF RESOLUTION

Armin HAKEN
Department of Computer Science, University of Toronto, Toronto, Ontario M5S 1A4, Canada

resolution exponential lower bound
classical separation result

Problem 1: swapping pigeons

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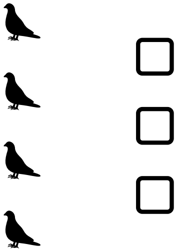

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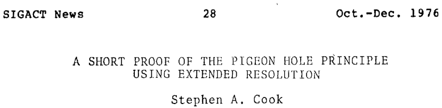
SIGACT News 28 Oct.-Dec. 1976

A SHORT PROOF OF THE PIGEON HOLE PRINCIPLE
USING EXTENDED RESOLUTION

Stephen A. Cook

extended resolution $O(n^4)$

new variables as definitions

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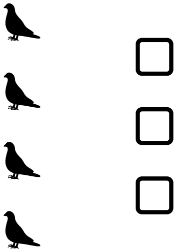

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Short Proofs Without New Variables

Marijn J.H. Heule[1(✉)], Benjamin Kiesl[2], and Armin Biere[3]

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DPR conditionally assigns variables w.l.o.g to $\top/\bot$

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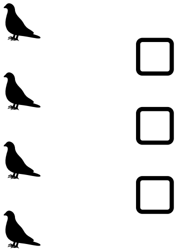

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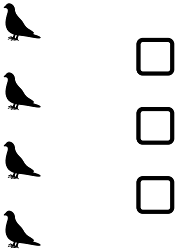

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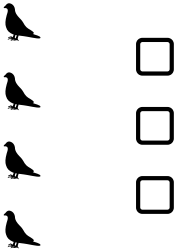

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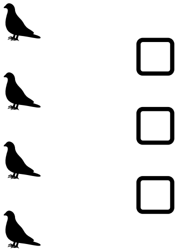

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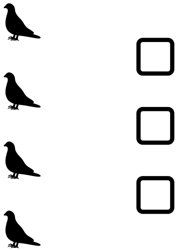

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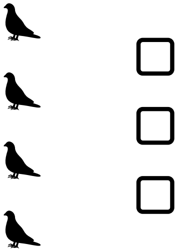

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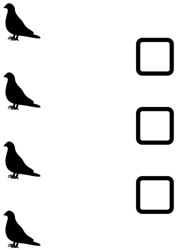

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why does this fail?

how do we make it work?

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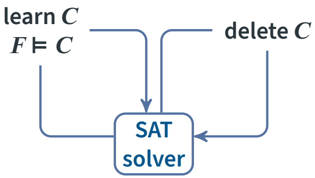

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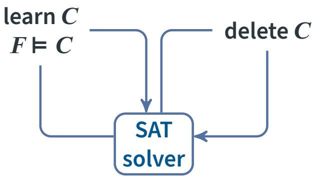

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(a form of iterated resolution + subsumption)
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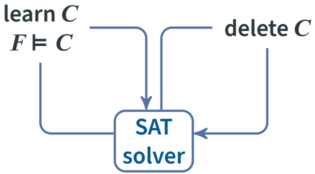

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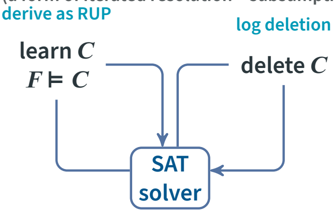

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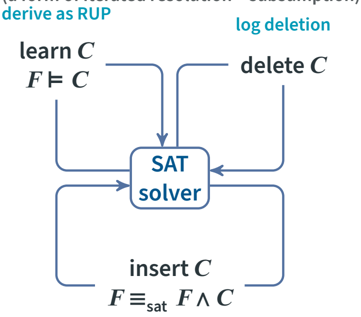

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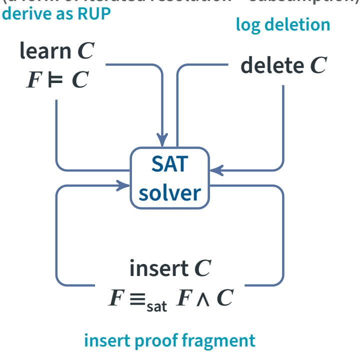

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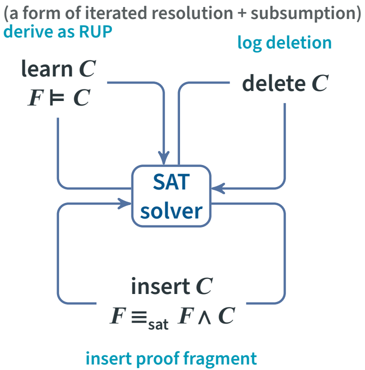


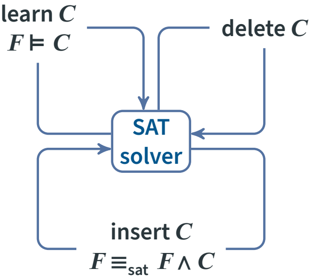
Proof generation for inprocessing

Problem 2: interference-free lemmas

(a form of iterated resolution + subsumption)

derive as RUP

log deletion

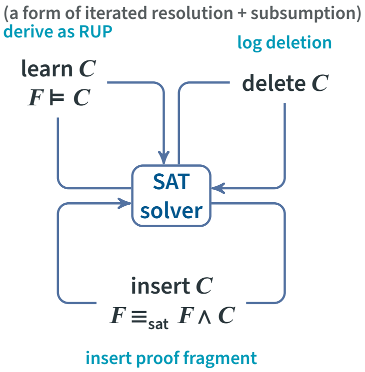


insert proof fragment

Proof generation for inprocessing

if C is a RUP over F

Problem 2: interference-free lemmas



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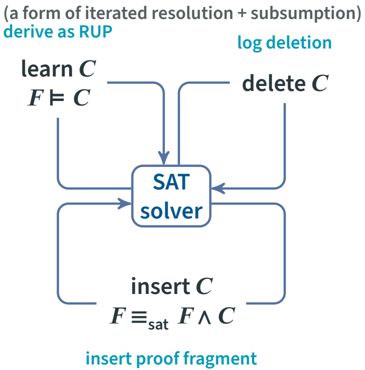

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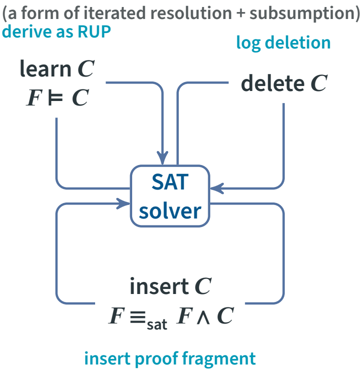

Proof generation for inprocessing

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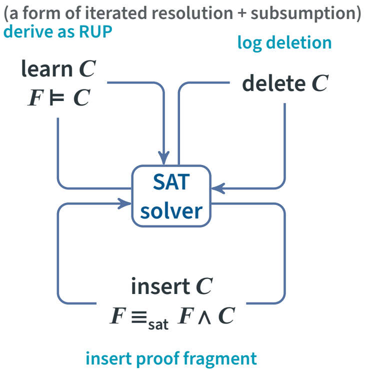

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i: L_1
i: L_2
i: L_3
i: C
d: L_3
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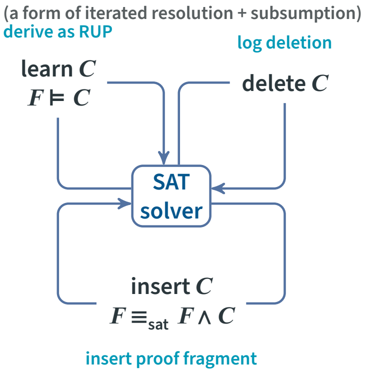

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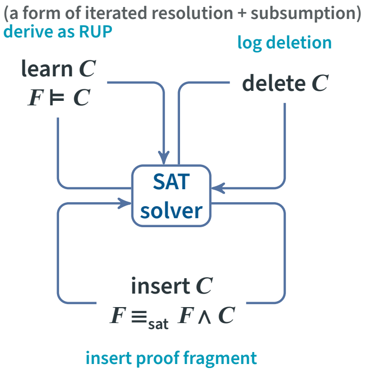

Proof generation for inprocessing

i: C

if C is an SR clause over F upon σ

C is an SR clause if $\sigma(C)$ is a tautology and all clauses in $\sigma(F)$ are RUP clauses over $F|_{\bar{C}}$

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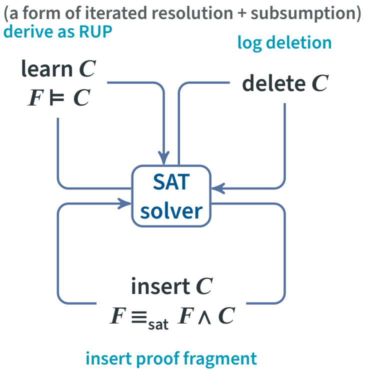

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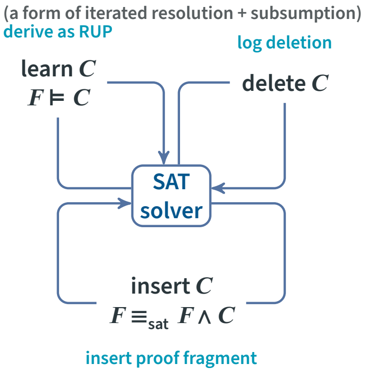

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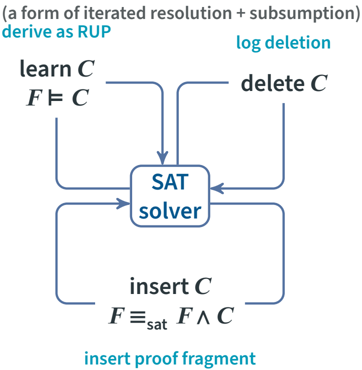

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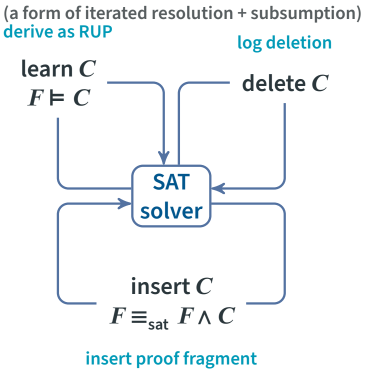

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i: L_1
i: L_2
i: L_3
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d: L_3
d: L_2
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Problem

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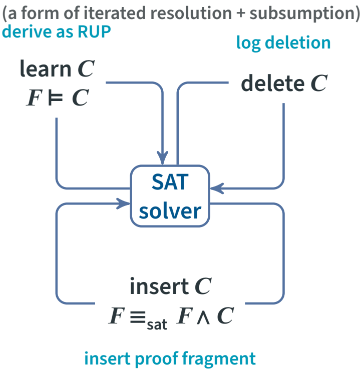

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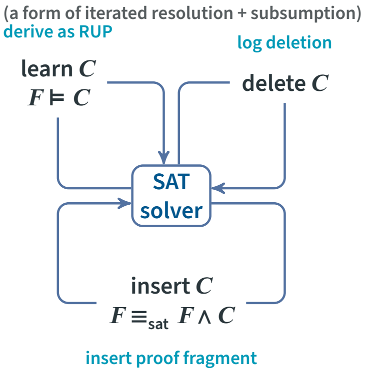

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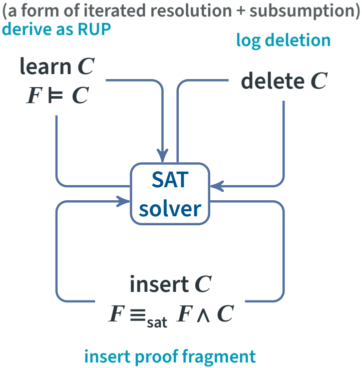

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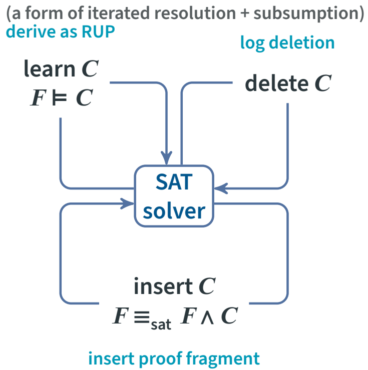

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how can we temporarily introduce interference-free lemmas?

Problem 3: cores and trimming under interference

Generating an unsatisfiable core from a proof

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mark the empty clause and proceed backwards

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$$\text{sub}\ \frac{E_0}{A_0} \qquad \text{res}\ \frac{A_0 \qquad E_1}{A_1} \qquad \text{res}\ \frac{A_1 \qquad E_2}{A_2} \qquad \ddots \qquad \text{res}\ \frac{A_{n-1} \qquad E_n}{C}$$

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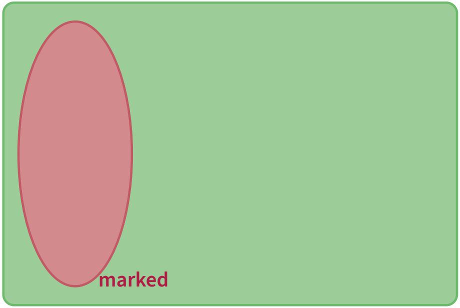

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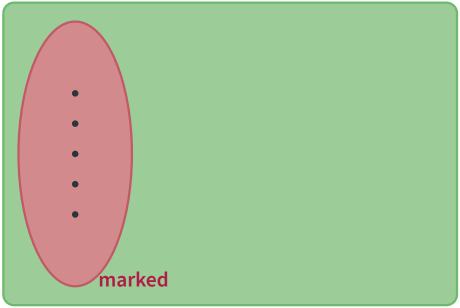

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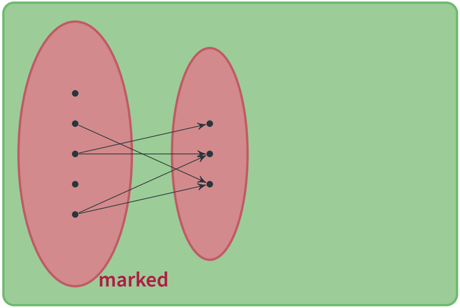

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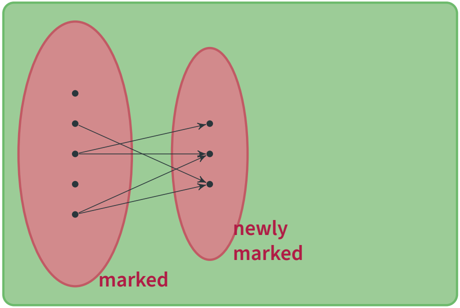

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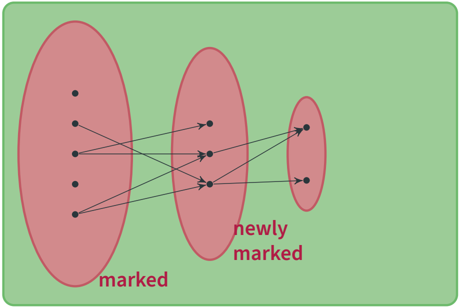

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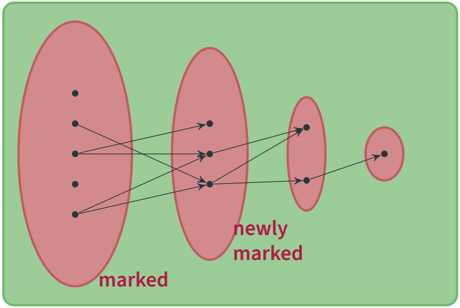

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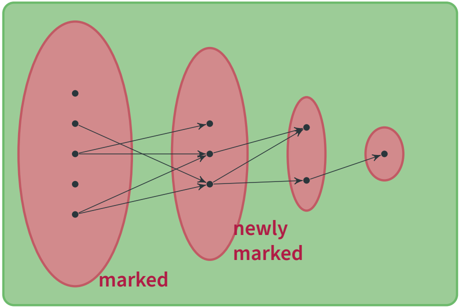

can we do better than this fixpoint computation?

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In this talk...

DRAT/DPR/DSR proofs

interference-free lemmas

pigeonhole refutation

non-fixpoint cores and trimming

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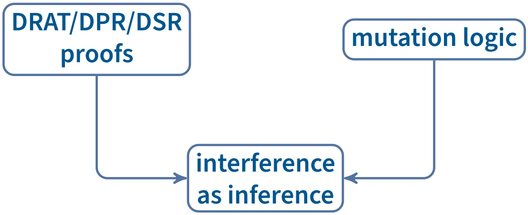

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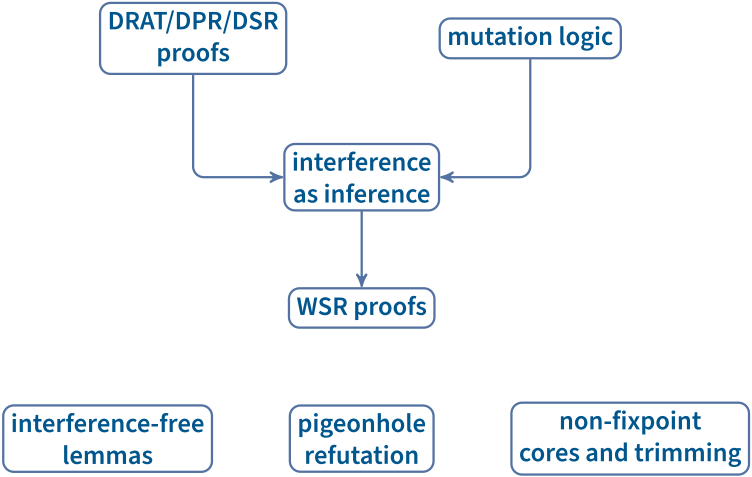
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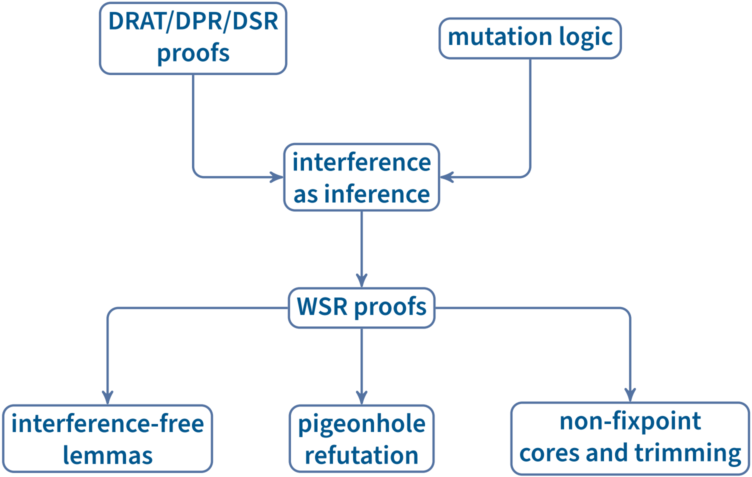

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The interference zoo

Propagation-based redundancy notions

The interference zoo

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RUP [Goldberg, Novikov ’03]

C is a reverse unit propagation (RUP) over F whenever unit propagation over $F \wedge \neg C$ reaches a conflict

The interference zoo

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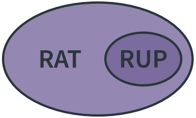

[Järvisalo, Heule, Biere ’12]

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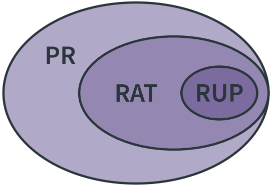

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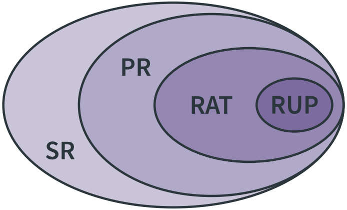

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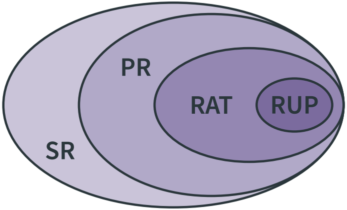

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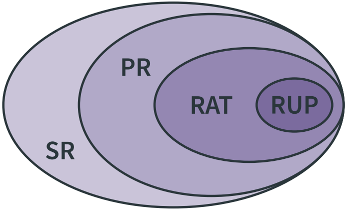

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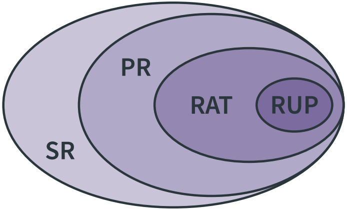

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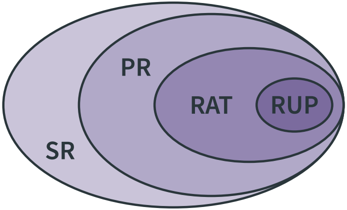

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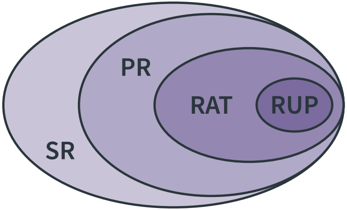

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can we relax the conditions for SR?

this extends work from [Rebola-Pardo, Suda ’18]

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substitutions $\sigma:$ variables $\to$ formulas

Mutation logic, or how I learned to stop worrying and love interference

this extends work from [Rebola-Pardo, Suda ’18]

Substitutions can be seen as transformations on interpretations

interpretations	$I :$	variables $\rightarrow \{0, 1\}$
		formulas $\rightarrow \{0, 1\}$
substitutions	$\sigma :$	variables $\rightarrow$ formulas
mutations	$I \circ \sigma :$	variables $\rightarrow \{0, 1\}$

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conditional mutations	$I \circ (\sigma :\!- Q) = \begin{cases} I \circ \sigma & \text{if } I \models Q \\ I & \text{if } I \not\models Q \end{cases}$

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$$\underbrace{I \circ (\sigma}_{\text{effect}} \;{:}{-}\; \underbrace{Q)}_{\text{trigger}} = \begin{cases} I \circ \sigma & \text{if } I \models Q \\ I & \text{if } I \not\models Q \end{cases}$$

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Mutation logic propositional logic + mutation operator

$$\nabla(\sigma :\!- Q).\, F$$

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Mutation logic   propositional logic + mutation operator

$$I \vDash \nabla(\sigma :\!- Q).\, F \quad \text{iff} \quad I \circ (\sigma :\!- Q) \vDash F$$

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Mutation logic propositional logic + mutation operator

$$I \vDash \nabla(\sigma :- Q).\, F \quad \text{iff} \quad I \circ (\sigma :- Q) \vDash F$$

Theorem if C is an SR clause over F upon σ, then $F \vDash \nabla(\sigma :- \bar{C}).F \wedge C$

Interference is inference (in mutation logic)

$$I \models \nabla(\sigma :- Q).\ F \quad \text{iff} \quad I \circ (\sigma :- Q) \models F$$

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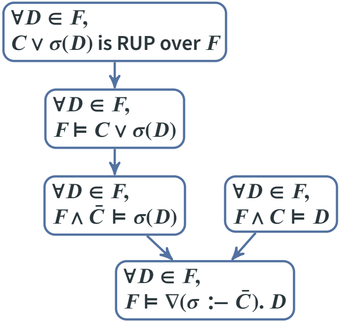

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$$I \vDash \nabla(\sigma \coloneq Q).\, F \quad \text{iff} \quad I \circ (\sigma \coloneq Q) \vDash F$$

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\boxed{\begin{array}{l}\forall D \in F, \\ F \wedge \bar{C} \vDash \sigma(D)\end{array}} \arrow[dr] & \boxed{\begin{array}{l}\forall D \in F, \\ F \wedge C \vDash D\end{array}} \arrow[d] & \\
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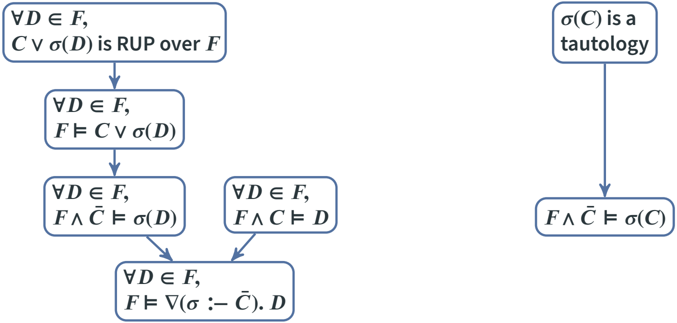

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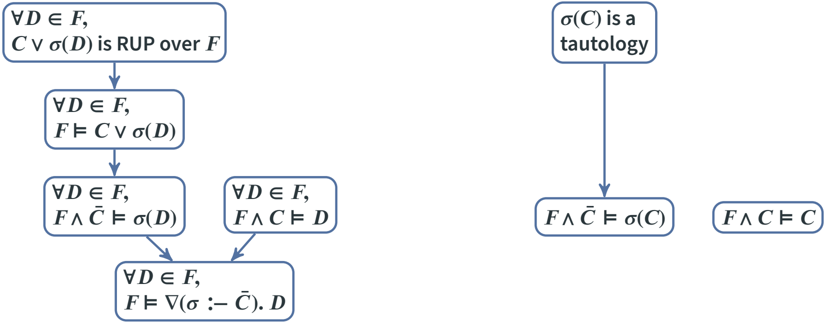

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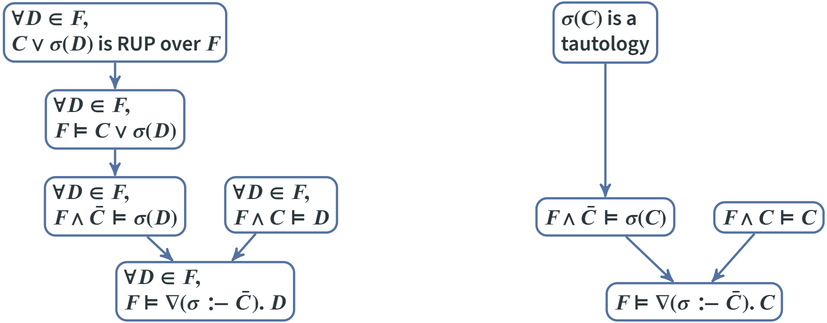

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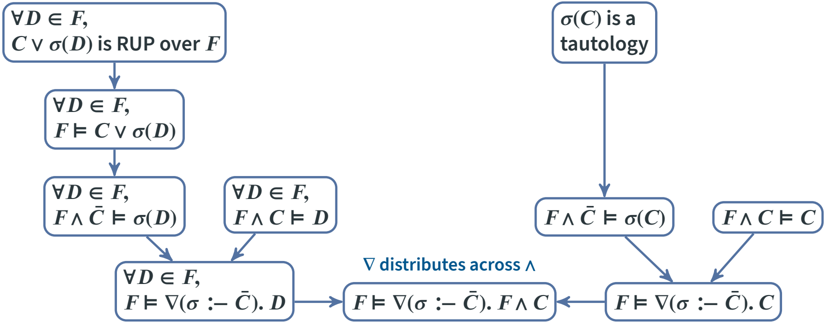

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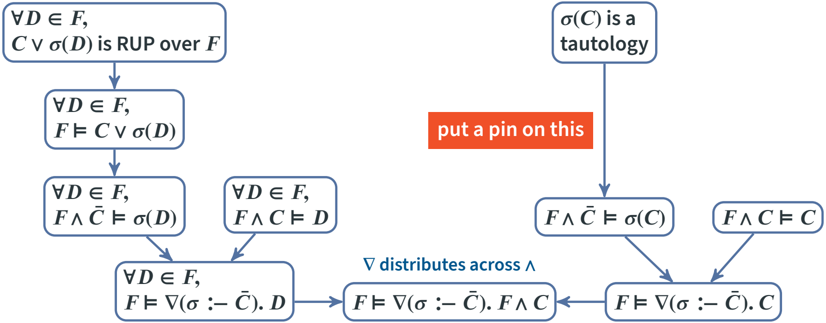

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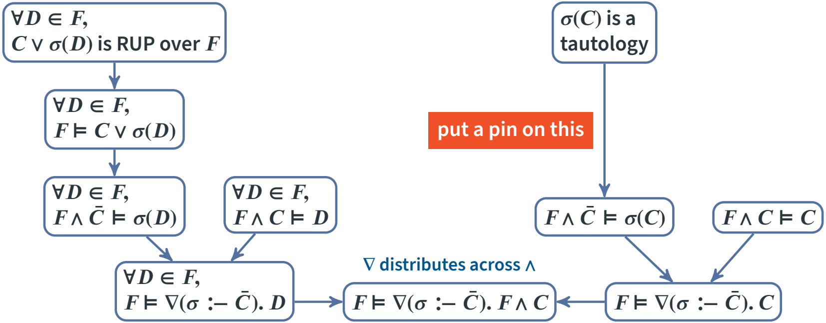

Interference is the same as deriving $\nabla(\sigma :\!- \bar{C}).D$ for each $D \in F \wedge C$

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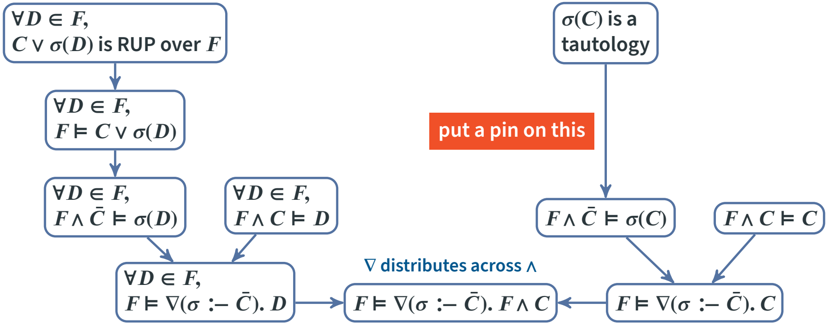
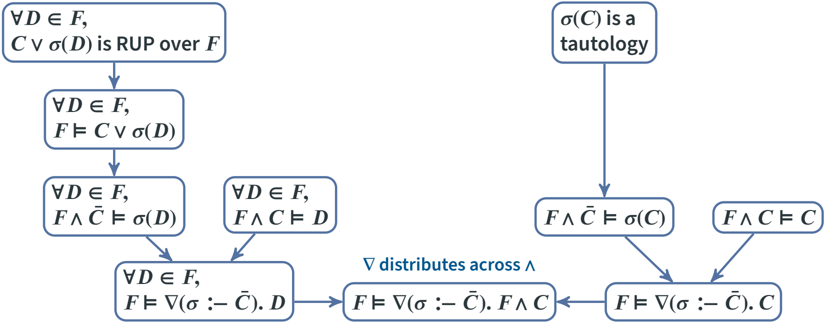


Interference is the same as deriving $\nabla(\sigma :\!- \bar{C}).D$ for each $D \in F \wedge C$

In the paper lots of details about mutation logic
+ a DAG-shaped proof system for interference!

I brought you some souvenirs from mutation world

Theorem if C is an SR clause over F upon σ, then $F \vDash \nabla(\sigma :- \bar{C}).F \wedge C$



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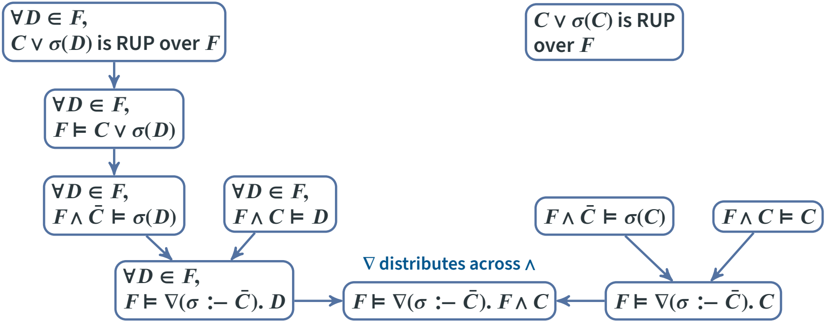

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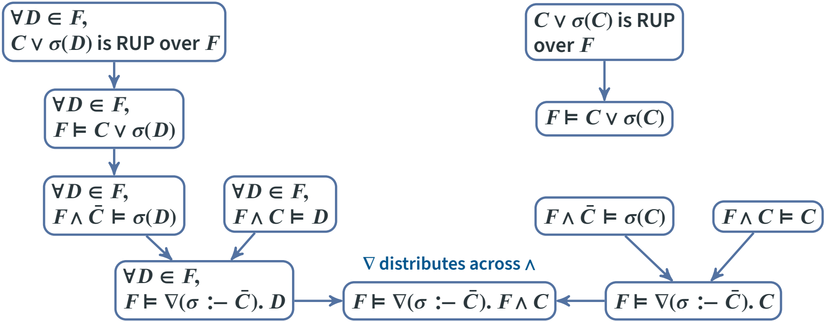

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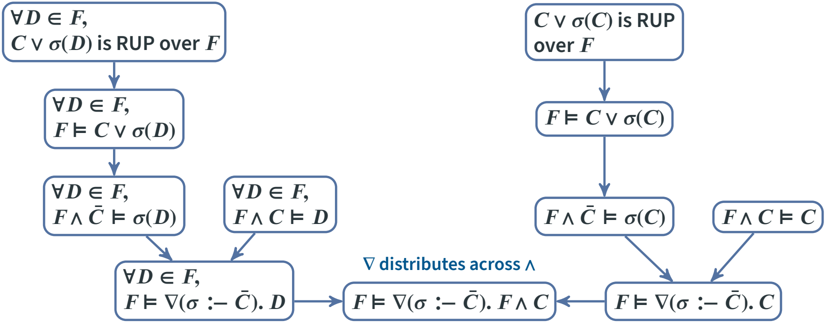

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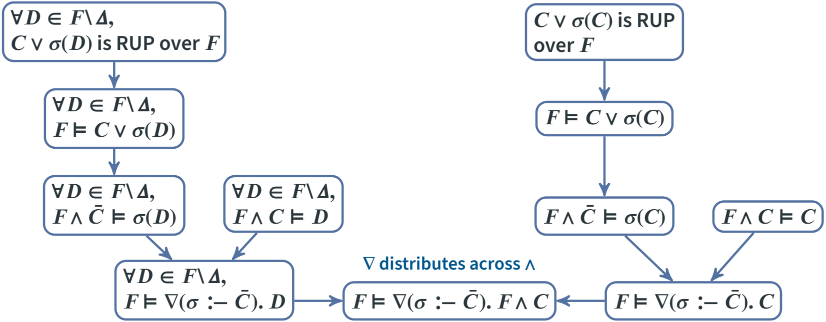

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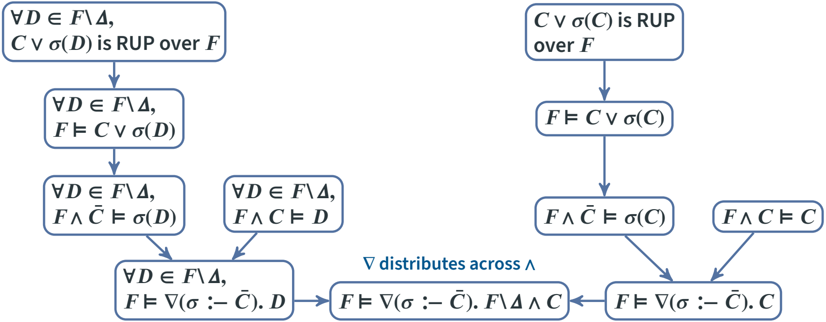

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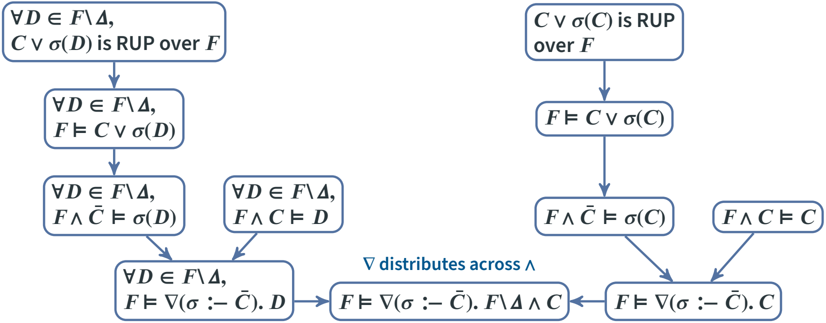

A clause C is a **weak substitution redundancy (WSR)** clause upon σ over F modulo Δ if, for all clauses $D \in F \setminus \Delta \wedge C$, the clause $C \vee \sigma(D)$ is a RUP clause over F.

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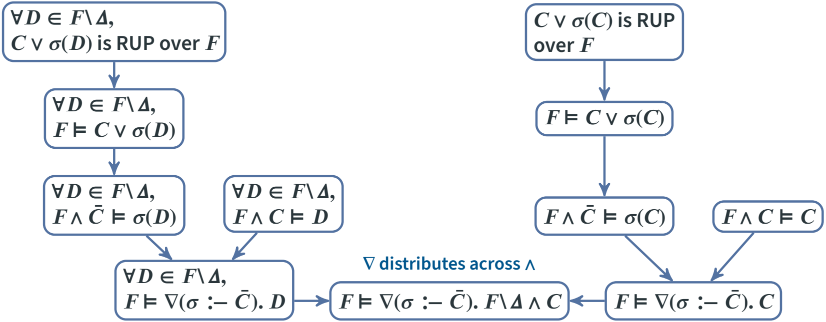

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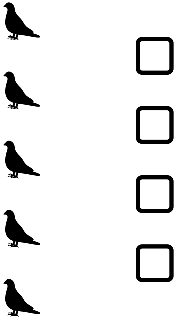
Theorem if C is a WSR clause over F modulo Δ upon σ, then
$F \vDash \nabla(\sigma :- \bar{C}).F \setminus \Delta \wedge C$

Swapping pigeons, finally

The pigeonhole problem $\mathrm{PHP}(n)$



Can we fit n pigeons into $n-1$ holes?

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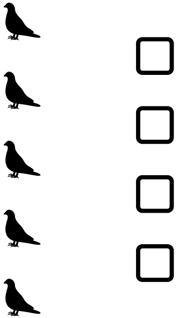

p_{ir} pigeon i is in hole r

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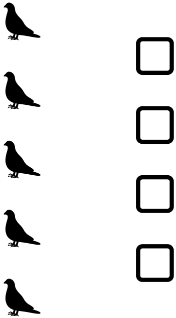

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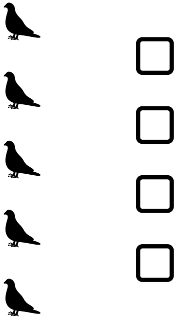

Can we fit n pigeons into $n-1$ holes?

- w.l.o.g. pigeon 1 is not in hole $n-1$

  otherwise swap pigeons 1 and n

  $\vdots$
- w.l.o.g. pigeon $n-1$ is not in hole $n-1$
- pigeon 1 is in some hole $1, \ldots, n-2$

  $\vdots$
- pigeon $n-1$ is in some hole $1, \ldots, n-2$
- solve $\mathrm{PHP}(n-1)$

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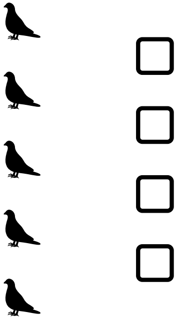

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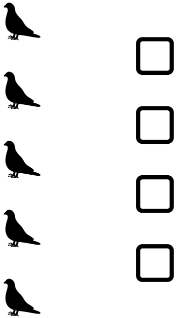

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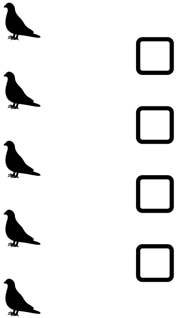

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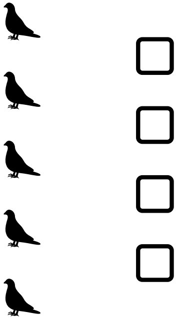

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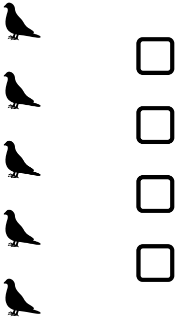

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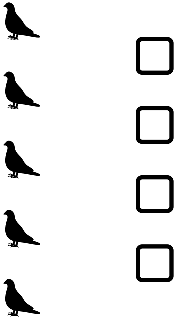
Swapping pigeons, finally

The pigeonhole problem $\mathrm{PHP}(n)$



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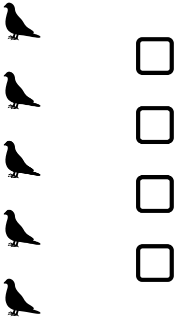
Swapping pigeons, finally

The pigeonhole problem $\mathrm{PHP}(n)$



Can we fit n pigeons into $n-1$ holes?

- w.l.o.g. pigeon 1 is not in hole $n-1$

  otherwise swap pigeons 1 and n

  introduce $C = \overline{p_{1(n-1)}}$ as WSR clause upon σ

  $\sigma = \{p_{1r} \mapsto p_{nr},\ p_{nr} \mapsto p_{1r} : 1 \le r < n\}$

  $C \vee \sigma(D)$ is RUP for each clause D!

  $C \vee \sigma(C) = \overline{p_{1(n-1)}} \vee \overline{p_{n(n-1)}}$ is RUP!

  … but it suffices that $C \vee \sigma(C)$ is a RUP

p_{ir} pigeon i is in hole r

$p_{i1} \vee \cdots \vee p_{i(n-1)}$ for $1 \le i \le n$

$\overline{p_{ir}} \vee \overline{p_{jr}}$ for $1 \le i < j \le n$ and $1 \le r < n$

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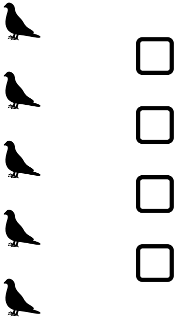

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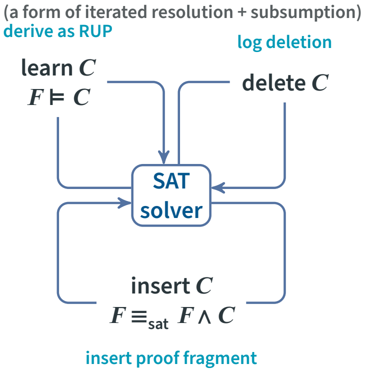
Interference-free lemmas, finally



Proof generation for inprocessing

i: L_1
i: L_2
i: L_3
i: $C\ [\sigma]$
d: L_3
d: L_2
d: L_1

all clauses in $\sigma(F)$ are RUP clauses over $F \wedge L_1 \wedge L_2 \wedge L_3$

if $F|_{\bar{C}} \vDash \sigma(F)$

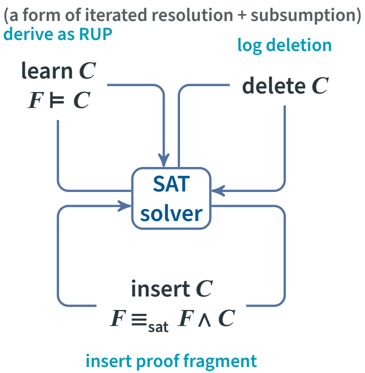
C is an SR clause if $\sigma(C)$ is a tautology and all clauses in $\sigma(F)$ are RUP clauses over $F|_{\bar{C}}$

we also need $\sigma(L_1), \sigma(L_2), \sigma(L_3)$ to be RUPs over $F \wedge L_1 \wedge L_2 \wedge L_3$

... which might need extra lemmas themselves...

... and so on...

Interference-free lemmas, finally



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C is a WSR clause modulo Δ if
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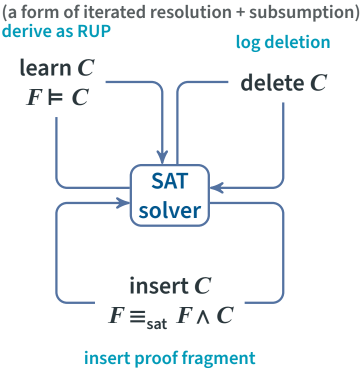

Proof generation for inprocessing

i: L_1
i: L_2
i: L_3
i: C $[\sigma]$ mod L_1, L_2, L_3

if $F|_{\bar{C}} \models \sigma(F)$

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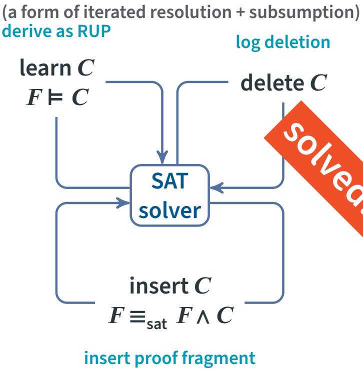

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Cores and trimming under interference, finally

Generating an unsatisfiable core from a proof

mark the empty clause and proceed backwards

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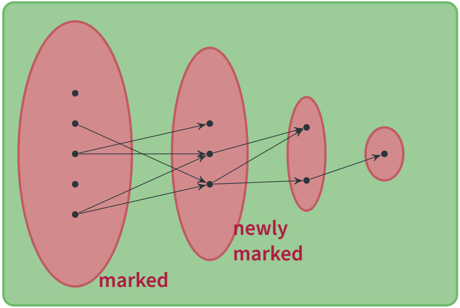
- if C is an input clause, it is in the core
- if C is a RUP clause, mark its antecedents
- if C is an SR clause upon σ...?



can we do better than this fixpoint computation?

for each marked D, $\sigma(D)$ is a RUP clause over $F|_{\bar{C}}$ $\Rightarrow$ mark their antecedents

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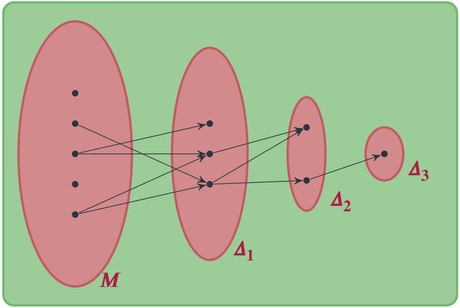

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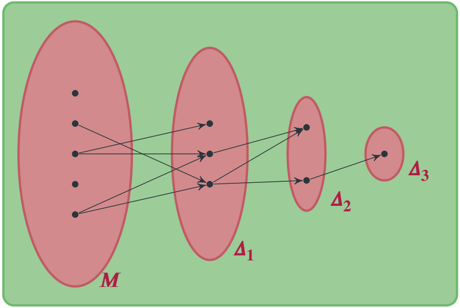

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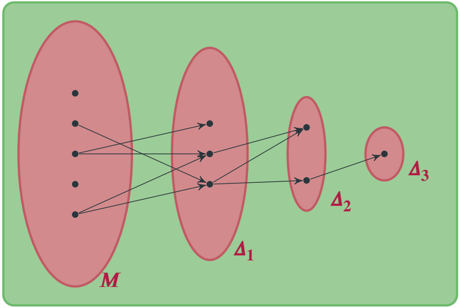

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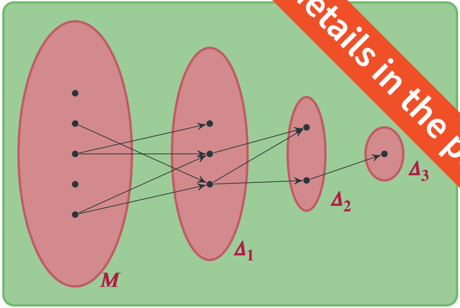

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... and maybe, potentially, perhaps, possibly, SMT/FOL?